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Lifelike Tessellations

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Although most people know of lifelike tessellations through the work of the Dutch artist M.C.Escher, (1898-1972) it often comes as a surprise to learn that he did not invent them and that they have their origins in Art Nouveau. Dense space filling designs can be seen in many patterns from this period, for example in the work of William Morris, several of whose fabrics patterns have closely packed leaves and flowers separated by a black line, with each leaf bending to fit its neighbour. This drawing by Nellie Syrett for the *Yellow Book* of 1896 is, if you like, a larval tessellation, insofar as a motif fills space and the background has vanished. It misses only because the motifs are read as overlapping rather than interlocking. It strikingly resembles the first tiling pattern which Escher drew, his "Eight Heads" of 1922. The first true lifelike tiling, where figure and ground become one, was probably 'Trout Dance" by Koloman Moser in the Viennese magazine *Ver Sacrum* in 1901, (4). It would have remained a curiosity had it not been for Escher, who made the subject his own by producing about one hundred and fifty of them during his life. He used them as raw material for his geometrical fantasy drawings and raised the art of inventing interlocking shapes to new and preposterous heights, (1). Drawing them became something of a mania for him. Since his time very few new tessellations have been published. Interested readers can find an entertaining pornographic example in issue 44 of the underground magazine *'OZ'* (1972), and three more, using bees, flowers and fish were produced in 1976 by Marjorie Rice, a San Diego housewife who had first made her name by discovering four hitherto unknown types of pentagons which tile the plane. Her patterns, which are based on the pentagons, and her achievements as an amateur mathematician are described in the book *'The Mathematical Gardner'(2).*

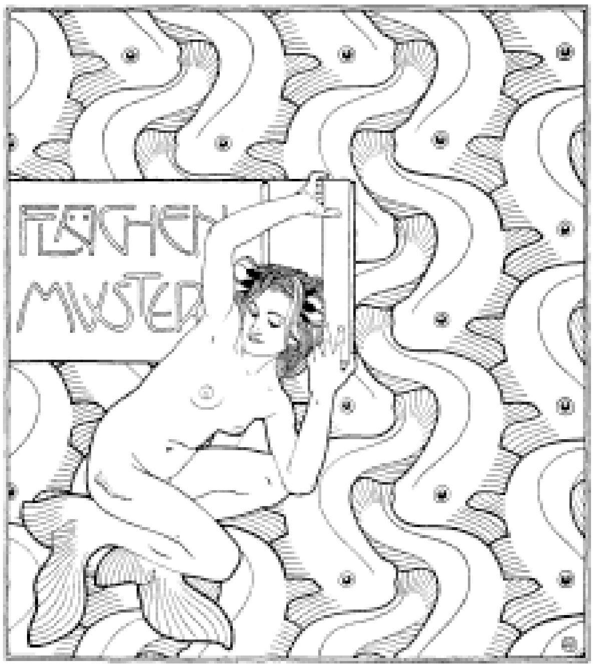

figure 1

Flächenkunst. by Kolo Moser 1901

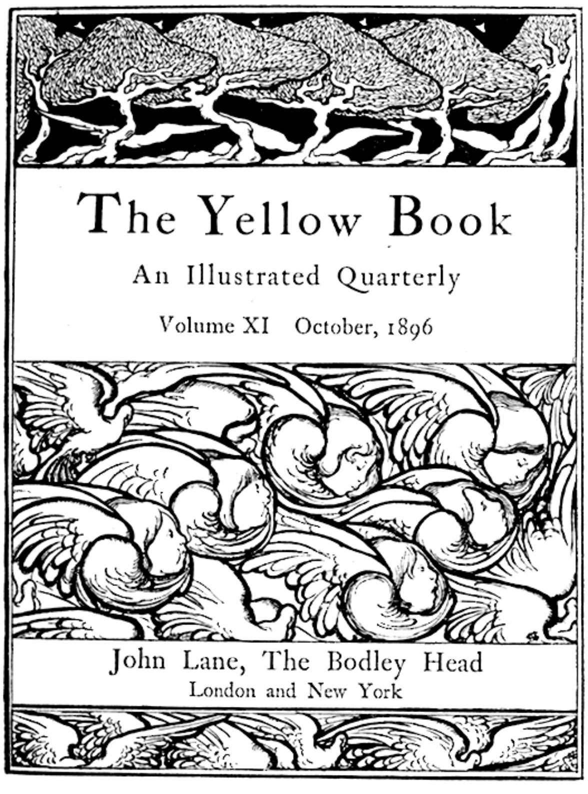

figure 2

The Yellow Book 1896, Frontispiece by Nellie Syrett

Aside from these there have been several books, mostly aimed at school children, giving instructions for drawing tessellations which approach the subject from a mathematical point of view, but the results are disappointing and are not to be compared with any of Escher's work. A search for people drawing tessellations, begun in 1990, in libraries, by advertising, and eventually on the internet has so far discovered 30 people in the world drawing them to a reasonable standard, examples of their work have been collected at , www.cromp.com/tess/home.html. Of these people Japanese artist Makato Nokamura's work is especially good.

It is trivially easy to construct shapes which will tile the plane but to draw one which looks like something can be difficult. To do it successfully involves a little geometry, a little art, and a lot of willpower. The results are often more like miniature works of engineering than drawings. The process is well suited to an architect since it resembles drawing a plan in that one has to continuously refine, adjust and make compromises between competing elements. A good tessellation has a slightly lunatic air, and ought to seem vaguely impossible, the important thing is that the perimeter of each tile has a double value, if this is not done properly the shapes will simply seem to overlap each other.

Figure 3

Badger line

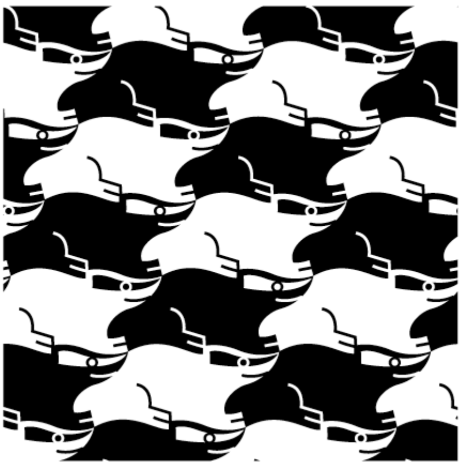

Figure 4

Badger Tiling

Tessellations are minimal drawings, for example the patterns of badgers shown here is formed by repeating this motif. It is difficult to imagine a badger drawn with a simpler line. This economy of expression is achieved making every point along its length have two meanings, one for each side of the line. This visual irony is the quality which brings the drawing to life.

One approach to drawing lifelike tilings is to use an algorithm to create tiles at random in the hope that one will be produced which looks like something. This crude method works surprisingly well, and is good for creating tilings which interlock shapes in unusual ways. The more usual method however, is to draw the thing to be tessellated then try to fit it to copies of itself using tracing paper. The process is made easier if it is realised that certain combinations are always impossible, for example, although four or six tiles may be fitted around a point, five never can. Although ordinary graphics software such as Freehand is useful for drawing the final pattern software for inventing the thing in the first place is generally an impediment. Some simple programmes for generating patterns automatically whilst working on a motif do exist but they obscure rather than illuminate the essential problem, which is working up and improving a simple line. This is an activity, like life drawing, in which using paper and a pencil is an advantage.

Although some of the earliest work on the mathematics of tilings was done by Kepler the subject was not taken completely seriously by mathematicians until fairly recently.It has now however, become mathematically respectable. Geometry, especially the study of fractals and tilings, has seen a revival in the last twenty years or so in no small part due to computer graphics which has enabled the study of patterns which would have been either impossible or unbearably tedious to draw by hand. The standard work on the mathematics of tilings is 'Tilings and Patterns" by Grunbaüm and Shepard (3), which draws on an unexpectedly rich field of ideas, including group theory, topology, and logic. The discovery of Penrose tilings which exist in a nether world between symmetry and asymmetry are one of the most exciting developments in the field.

Figure 5

Isohedral pattern of birds

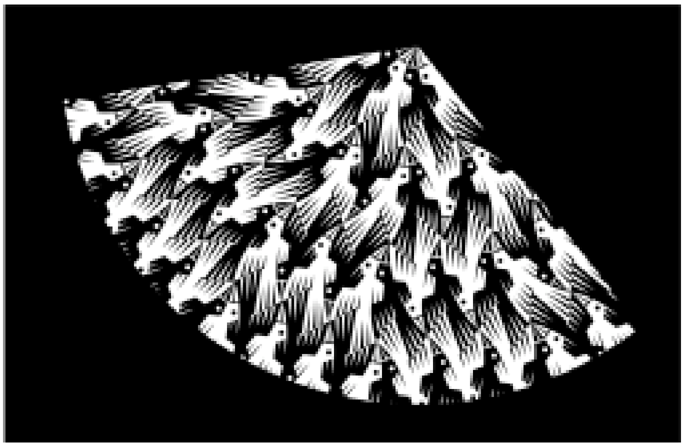

Figure 6

Anisohedral pattern of birds

Tilings are conveniently divided into two types, isohedral and anisohedral. Sometimes a shape can form patterns of both types, the birds shown here are isohedral when they are tiled in rows and anisohedral where they grow from a point. In an isohedral pattern every tile is identically situated, in anisohedral patterns a tile can occur in different ways in the pattern, in this case either near or far from the centre. Isohedral patterns are, mathematically speaking, well behaved and well understood, the remaining, anisohedral tilings which include such oddities as Penrose tilings, and Voderberg spirals (3), are not so easy to classify and enumerate. All Escher's patterns, in fact nearly all patterns in everyday use, are isohedral or have an isohedral repeating unit, not least because they can be printed from a roller. Anisohedral patterns have hardly been discovered by designers, even though their controlled irregularity offers some challenging opportunities.

Lifelike tessellations are based on patterns of polygons with their sides replaced by motifs, the pattern of badgers for example was formed by substituting the sides of an parallelogram. This is the simplest algorithm but there are many others, is it possible to list all them all? If we restrict ourselves to isohedral patterns the answer to this question is yes, in 1977 Grunbaum and Shepard's nineteen page proof showed that there are only 81 isohedral types tilings of the plane to which they named IH1 to IH81. Each of the isohedral types may be used as an algorithm for drawing tessellations. The badgers shown here belongs to type IH41.The total of 81 types included patterns which contain straight edges, for example one of the types is a pattern of squares in its own right, and if those are set aside there remain forty nine ways of drawing tilings where you are free to alter the entire perimeter of each tile. An abstract example is each of the forty nine types is shown here.

IH41	IH1	IH84	IH46	IH47	IH57	IH23
IH4	IH8	IH33	IH7	IH10	IH36	IH18
IH43	IH44	IH2	IH3	IH86	IH51	IH52
IH53	IH59	IH25	IH27	IH5	IH6	IH9
IH79	IH55	IH61	IH62	IH28	IH71	IH73
IH39	IH88	IH90	IH31	IH34	IH11	IH21
IH66	IH69	IH15	IH13	IH68	IH12	IH74

Figure 7

Key to forty nine ways of drawing lifelike tessellations

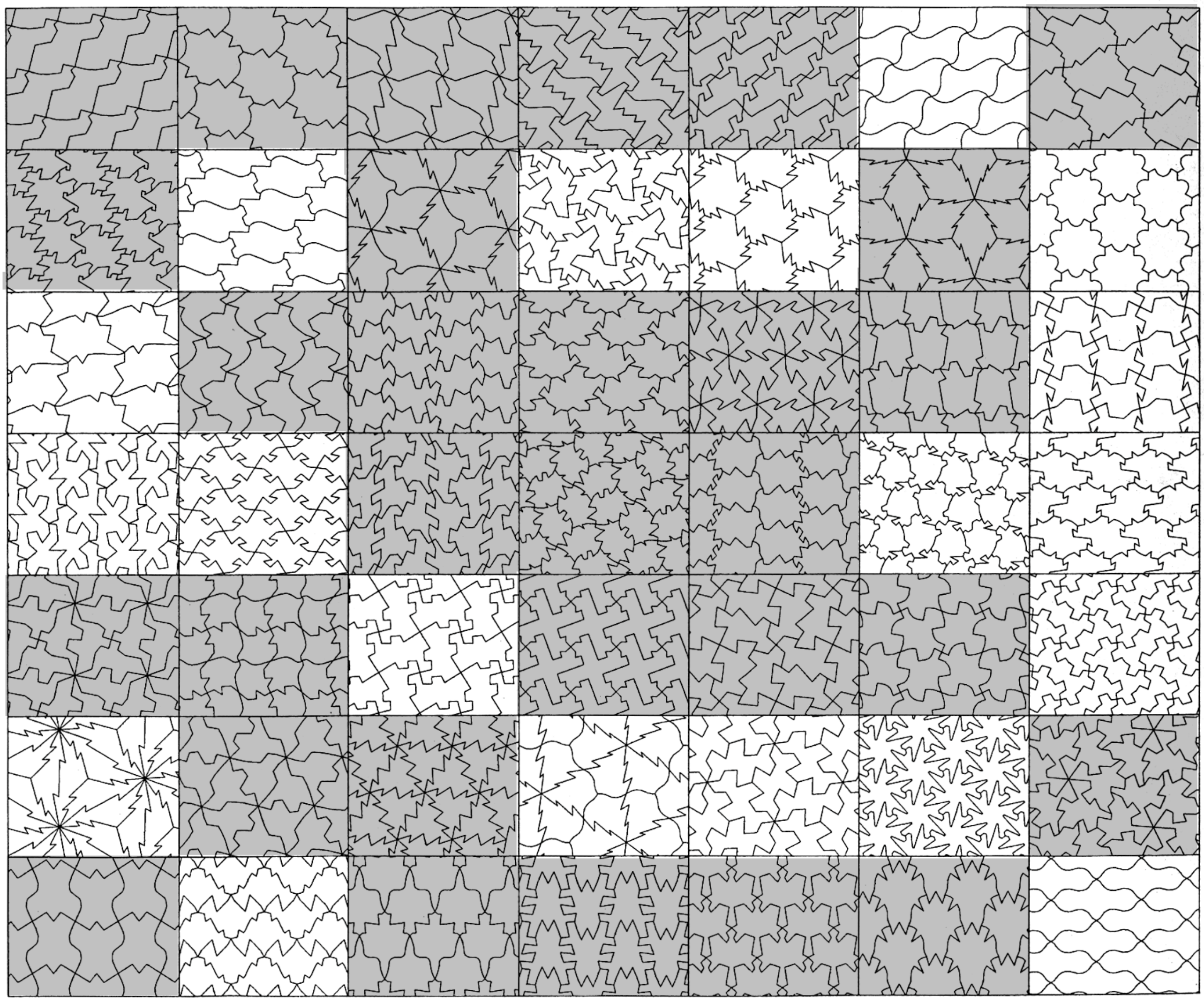

Figure 8

Forty nine isohedral types of tiling the plan

There is a strange charm to this diagram, all the tilings are as fundamentally different as are the elements in the periodic table, it shows the fundamental limits on the ways that a shape may be combined with itself to cover the plane. None could be smoothly morphed into another without going through some intermediate form which is simpler than either of them. Those which have at one time or another been used for lifelike tilings are shown shaded so the number of white cells indicates that many ways of combining shapes in lifelike tessellations are as yet unexplored. It is common enough for architects and designers to divide up space in a regular way, this diagram shows that there are many ways of doing this which are unsuspected and unexplored.

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