Wave-Particle Duality: de Broglie Waves and Uncertainty

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Abstract In 1925, de Broglie hypothesized that any material particle has an associated wave with $\lambda = h/p$. Electron diffraction seems to support that Hypothesis. But then, the electron at rest will have infinite wavelength, and infinite wave phase velocity. This says that for a material particle, the de Broglie relation does not hold.

Failed attempts to save the postulate, kept the flawed relation, and modified the waves into train waves, pilot waves, probability waves,… to name a few. We keep the de Broglie waves unchanged, and modify the relation.

First, we observe that the Planck energy $E = h\nu$ used by de Broglie defines virtual electromagnetic waves.

Consequently, for any particle, the virtual electromagnetic wavelength is $\lambda = c/v = h/mc$, and $h/m\upsilon = \lambda c/\upsilon$.

Refinement of de Broglie argument, indicates that $h/m\upsilon$ may be Δx , Heisenberg's uncertainty in the particle location.

de Broglie's later analysis supports this interpretation, and we offer an explanation to particle diffraction as a consequence of Heisenberg's uncertainty.

We apply $h/m\upsilon = \lambda c/\upsilon$ to obtain the dispersion relations for the de Broglie virtual waves.

Finally, we observe that reinstating the matter wave, String theory avoids Uncertainty.

1. de-Broglie waves

de-Broglie [1] associated with a particle at rest an internal rest frequency v_0 defined by

$$hv_0 = m_0 c^2. \qquad (1)$$

If the particle has speed υ, denote

$$\gamma \equiv \frac{1}{\sqrt{1-(\upsilon^2/c^2)}}$$

Then, de Broglie assumed that the particle constitutes a plane wave propagated along the x-axis with frequency v defined by

$$hv = mc^2. \qquad (2)$$

That is,

$$hv = \gamma m_0 c^2 = \gamma hv_0$$

$$v = \gamma v_0$$

The variation of the phase of the wave over the time interval

$$\delta t = \frac{\delta l}{\upsilon}$$

is

$$\delta\varphi = \omega\delta t - k\delta l$$

$$= 2\pi v\delta t - \frac{2\pi}{\lambda}\upsilon\delta t$$

$$= 2\pi\delta t(\gamma v_0 - \frac{\upsilon}{\lambda}), \qquad (3)$$

where, it is understood, that λ stands for the wavelength of the so-called de Broglie matter wave.

On the other hand, by the clock retardation formula, the observed internal frequency of the particle is

$$v_i = v_0 / \gamma,$$

and the observed variation in the time interval δt of the internal phase of the particle is

$$\delta \varphi_i = (2\pi\delta t)v_0 / \gamma.$$

To have the particle remain incorporated in its wave, de Broglie set

$$\delta \varphi = \delta \varphi_i. \quad (4)$$

That is,

$$\gamma v_0 - \frac{\upsilon}{\lambda} = v_0 / \gamma$$

$$v_0 \gamma - v_0 / \gamma = \frac{\upsilon}{\lambda}$$

$$\gamma h v_0 (1 - 1/\gamma^2) = h\frac{\upsilon}{\lambda}$$

$$\gamma m_0 c^2 (1 - [1 - \upsilon^2 / c^2]) = h\frac{\upsilon}{\lambda}$$

$$m\upsilon^2 = h\frac{\upsilon}{\lambda}$$

$$\lambda = \frac{h}{m\upsilon} \qquad (5)$$

The term $\dfrac{h}{m\upsilon}$ is measured in electron diffraction, but equating it to λ leads to an infinite phase velocity of the wave associated with an electron at rest. Indeed, for an electron at rest, we have

$$\upsilon = 0,$$

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$$\lambda_0 = \infty.$$

Now, by (1),

$$v_0 = m_0 c^2 / h > 0.$$

Therefore, the de Broglie wave phase velocity for an electron at rest is

$$\lambda_0 v_0 = \infty(m_0 c^2 / h) = \infty. \qquad (6)$$

This non-physical result points to an error in de Broglie relation (5).

For a material particle, the relation (5) does not hold.

Where did de Broglie go wrong?

We proceed to observe that de Broglie's use of the photon energy, $E = h\nu$ implies electromagnetic waves.

2. $E = h\nu$ mandates virtual Electromagnetic waves.

By assuming the equation

$$mc^2 = h\nu \qquad (2)$$

de Broglie assumed two equations:

Planck's equation for the energy of the photon,

$$E = h\nu, \qquad (7)$$

and Einstein's equation for the energy of a particle

$$E = mc^2. \qquad (8)$$

A photon is electromagnetic radiation energy, with wavelength

$$\lambda = c / v. \qquad (9)$$

If equation (2) is used

$$\lambda = h / mc. \qquad (10)$$

The electromagnetic radiation problem of the 1900's, that was resolved with Planck's equation,

$$E = h\nu$$

boiled down in later generations to a particle equation, and the electromagnetic waves were forgotten.   But invoking Planck's equation means assuming electromagnetic radiation.

Electromagnetic radiation has speed c, wavelength λ, and frequency ν. that satisfy the relation (9), which gives the dispersion relation

$$\omega = 2\pi\nu = c2\pi / \lambda = ck$$

de Broglie assumption of

$$\lambda = h / m\upsilon \ ,$$

leads to an infinite phase velocity of the virtual wave associated with an electron at rest because by (10)

$$\lambda = h / mc \ .$$

In order to resolve the electromagnetic radiation spectrum problem, Planck's hypothesized that electromagnetic radiation energy exists in discrete packets of

$$E = h\nu \ .$$

Einstein emphasized the particle nature of the electromagnetic energy, and the Compton effect, and the Bohr atom amplified the particle nature of light even further.

But the photon equation (7) describes precisely, and exclusively electromagnetic radiation.

A non-electromagnetic wave, as de Broglie thought of his wave, cannot satisfy the Planck equation that characterizes uniquely electromagnetic radiation energy.

If Planck's equation is assumed, the wave associated with the particle must be electromagnetic. Not a matter wave, Not a pilot wave.

Consequently, the wavelength of the virtual electromagnetic wave can be only

$$\lambda = \frac{c}{v}.$$

This resolves the problem of infinite phase velocity of the virtual wave associated with an electron at rest.

de Broglie analysis leads to the term $\frac{h}{m\upsilon}$ that measures electron diffraction, but $\frac{h}{m\upsilon}$ does not equal the virtual wavelength $\lambda = \frac{h}{mc}$.

What is it equal to?

Can we replace the incorrect relation (5) with a physically believable relation?

3. The de Broglie Relation, and Uncertainty

Since

$$\lambda = \frac{c}{v} = \frac{h}{mc}, \qquad (11)$$

the de Broglie term that is measured in particle diffraction is

$$\frac{h}{m\upsilon} = \lambda \frac{c}{\upsilon}. \qquad (12)$$

If the particle is a photon, then

$$\upsilon = c,$$

and the de Broglie term equals to the wavelength λ, which measures the uncertainty in the x location of the photon, Δx.

If the particle is an electron, photon diffraction suggests interpreting the de Broglie term of electron diffraction as the uncertainty in the particle x location, Δx.

We show that a modified de Broglie analysis points to the relation

$$\Delta x = \lambda \frac{c}{\upsilon} = \frac{h}{p},$$

where Δx denotes the uncertainty in the location of the particle.

We have

$$v = \frac{c}{\lambda}$$

$$v \frac{\upsilon^2}{c^2} = \frac{\upsilon^2}{\lambda c}$$

$$\gamma v_0 (1 - [1 - \upsilon^2 / c^2]) = \frac{\upsilon^2}{\lambda c}$$

$$\gamma v_0 (1 - 1/\gamma^2) = \frac{\upsilon^2}{\lambda c}$$

$$\gamma v_0 - \frac{\upsilon^2}{\lambda c} = v_0 / \gamma$$

$$2\pi \delta t \left(\gamma v_0 - \frac{\upsilon}{\lambda c / \upsilon} \right) = 2\pi \delta t \left(v_0 / \gamma \right) \quad (13)$$

Equation (13) says that the particle remains incorporated in its uncertainty zone because

$$\delta\varphi = \delta\varphi_i . \quad (14)$$

As in de Broglie analysis,

$$\delta\varphi_i = (2\pi \delta t) v_0 / \gamma$$

is the variation in the time interval

$$\delta t = \frac{\delta l}{\upsilon}$$

of the internal phase of the particle, since by the clock retardation,

$$v_i = v_0 / \gamma .$$

Now,

$$\delta\varphi = 2\pi\delta t(\gamma v_0 - \frac{v}{\lambda c / v})$$

$$= \omega\delta t - \frac{2\pi}{\lambda c / v}\delta l \quad (15)$$

is the variation of the phase of the virtual electromagnetic wave over the time interval δt.

If the particle is a photon, then

$$v = c ,$$

the wavenumber is

$$k = 2\pi / \lambda ,$$

and the uncertainty in the particle's x location is the wavelength λ.

If the particle is an electron, then

$$\frac{2\pi}{\lambda c / v}$$

functions as an effective wavenumber, and

$$\lambda\frac{c}{v} \qquad (16)$$

functions as an effective wavelength.

Photon diffraction suggests that the uncertainty in the particle's x location, equals the effective wavelength (16). That is,

$$\Delta x = \lambda\frac{c}{v} \qquad (17)$$

$$= \frac{hc}{mc^2}\frac{c}{v}$$

$$= \frac{h}{mv} \qquad (18)$$

It seems that the de Broglie term

$$\frac{h}{p}$$

that equals

$$\lambda \frac{c}{\upsilon}$$

determines the uncertainty in the location of the particle.

We are inclined to replace the de Broglie relation

$$\lambda = \frac{h}{m\upsilon},$$

with the relation

$$\Delta x = \frac{h}{m\upsilon}, \qquad (19)$$

where Δx denotes the uncertainty in the location of the particle.

If the particle is a photon, then

$$\Delta x = \lambda \qquad (20)$$

If the particle is an electron, then

$$\Delta x = \lambda \frac{c}{\upsilon} \qquad (21)$$

is strictly greater than the virtual electromagnetic wavelength.

Relation (19) is consistent with Heisenberg's uncertainty relations. Clearly,

$$(\Delta x)(\Delta p) = h > \frac{h}{2\pi} = \hbar. \qquad (22)$$

In particular, if

$$\upsilon = 0,$$

then,

$$\Delta p = 0,$$

and

$$\Delta x = \infty. \qquad (23)$$

We proceed to establish relation (19).

4. de Broglie later analysis and Uncertainty

For Schrodinger's wave function $\psi(x,t)$ for a particle [2], the Gaussian distribution

$$\left|\psi(x)\right|^2 = \frac{1}{\sqrt{2\pi}\,\Delta x}\exp\left(-\frac{1}{2(\Delta x)^2}x^2\right) \qquad (24)$$

measures the probability to find the particle at x, within uncertainty interval of size Δx.

It is well known [3] that the mean value of x is

$$\overline{x} = \int\limits_{x=-\infty}^{x=+\infty} x\left|\psi(x)\right|^2 dx = 0,$$

and the variation of x is

$$\sigma_x^2 = \overline{\left(x-\overline{x}\right)^2} = \overline{x^2} = \int\limits_{x=-\infty}^{x=+\infty} x^2\left|\psi(x)\right|^2 dx = \left(\Delta x\right)^2. \qquad (25)$$

By [4, p.20], the Fourier Transform of the Schrodinger wave function $\widehat{\psi}$, is a wave function for the momentum p.

By [4, p. 92], since $\left|\psi(x)\right|^2$ is Gaussian, then $\left|\widehat{\psi}(p)\right|^2$ is Gaussian,

$$\left|\widehat{\psi}(p)\right|^2 = \frac{1}{\sqrt{2\pi}\,\Delta p}\exp\left(-\frac{1}{2(\Delta p)^2}x^2\right) \qquad (26)$$

and

$$\sigma_p = \Delta p \qquad (27)$$

In [4, pp. 95-99], de Broglie seeks the range of the Heisenberg uncertainty intervals. de Broglie argues that in practice

$$\left(\Delta p\right)\left(\Delta x\right) \geq h. \qquad (28)$$

and in fact

$$(\Delta p)(\Delta x) \approx h. \qquad (29)$$

In de Broglie words [4, pp. 98-99]:

"In order to precisely define the Heisenberg uncertainties, it is necessary to define the uncertainty δA of any quantity A, as the interval of A values such that the probability of finding a value outside δA is less than some small quantity ε.

For two canonically conjugate quantities A, and B, one then finds that

$$(\delta A)(\delta B) \sim \alpha(\varepsilon)h$$

where $\alpha(\varepsilon)$ depends on the choice of ε.

the function $\alpha(\varepsilon)$ is infinite for $\varepsilon = 0$, so that then $(\delta A)(\delta B) = \infty$, from which it follows that δB is infinite if δA is finite (case of a sharp edged interval)

However, in practice it suffices to choose ε very small, <u>but nonzero</u>, and then the product $(\delta A)(\delta B)$ can in the most favorable cases become as small as something of the order h, but not smaller.

Practically, we thus have

$$(\delta A)(\delta B) \geq h$$

in order of magnitude."

The examples that de Broglie uses to illustrate the above, satisfy (29). In particular, for

$$\Delta p = m\upsilon$$

we have

$$\Delta x \approx \frac{h}{m\upsilon}.$$

we see that de Broglie later analysis points to uncertainty as the essence of his relation, and supports replacing his relation $\lambda = h/m\upsilon$ with a modified uncertainty relation
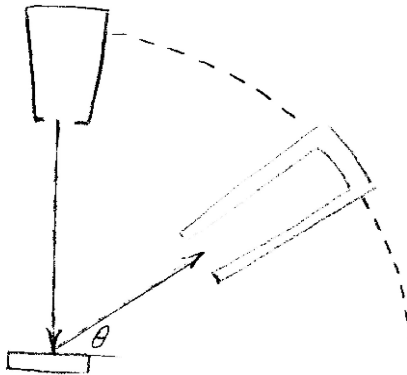
$$\Delta x = \frac{h}{m\upsilon}$$

5. Electrons and Uncertainty

In the Davisson-Germer experiment [5], electrons with energy of 25-350 electron-Volts were shot at a Nickel crystal.

The electrons that reflected elastically from the crystal, at an angle θ, and lost only a little of their energy, were allowed into a detector.
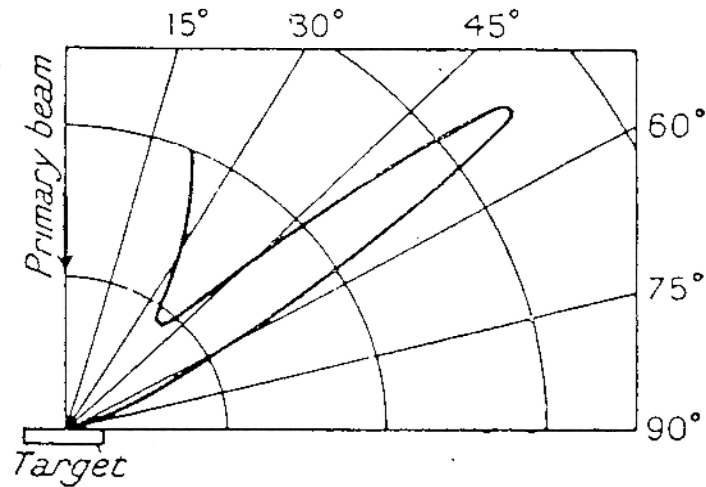
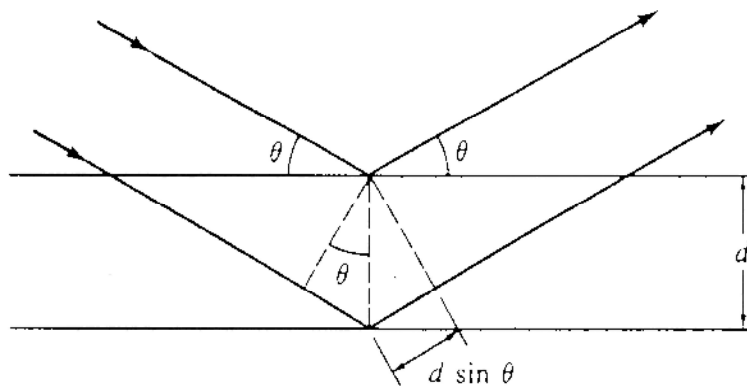The detector could be moved along a track, so that reflection would come from various crystal planes.

The electrons were reflected at angles

$$20° \leq \theta \leq 90°.$$

The electrons' distribution with respect to the angle, peaked at 54Volts, at an angle of $50°$.

As in Bragg diffraction, the particles are reflected at an angle θ, from two planes of atoms in a crystal that are spaced at distance d.

The path difference between successive reflections is

$$2d\sin\theta$$

Since the particles are distributed normally over the uncertainty interval $\Delta x = h/m\upsilon$, minimal number of particles are likely to be at the edges of the interval, and maximal number of particles are likely to be at the middle of the interval.

Therefore, maximal number of particles will aggregate in a direction θ
so that

$$2d\sin\theta = n\Delta x. \qquad (30)$$

Since the Fourier transform of a Gaussian curve is Gaussian, the
distribution of the reflected particles in the direction θ, remains
Gaussian.

6. X-rays and Uncertainty

X-rays were produced first by Rontgen in 1895, as the result of
decelerating electrons in a metal plate target. X-rays were not
deflected by electric and magnetic fields, and were determined to carry
no charge. Thus, they were suspected to be electromagnetic radiation.

Because of their energy, their frequency would be high, their wave
length short, and diffraction would require gratings with short
distance. The planes of atoms in a crystal are spaced at such short
distance, and can serve as diffraction gratings.

The suggestion to use crystals, was made by Max Von Laue in 1912.
The experiments by Friedrich and Knipping yielded uncertainties of
$(0.1-0.5)10^{-10} meter$, that were interpreted as wavelengths, in Bragg
diffraction of waves.

But the uncertainty need not be wavelength, Bragg diffraction applies
also to particles with $\Delta x = 2d\sin\theta$, and the diffraction experiments did
not prove that X-rays are photons.

X-rays did prove to be electromagnetic radiation in two other
experiments:

The first is the X-ray spectrum that is cut off at a given voltage V. The cutoff may be explained as a reversed photoelectric effect in which an electron with energy eV comes to a complete stop, and its kinetic energy eV converts in full to an X-ray photon of frequency
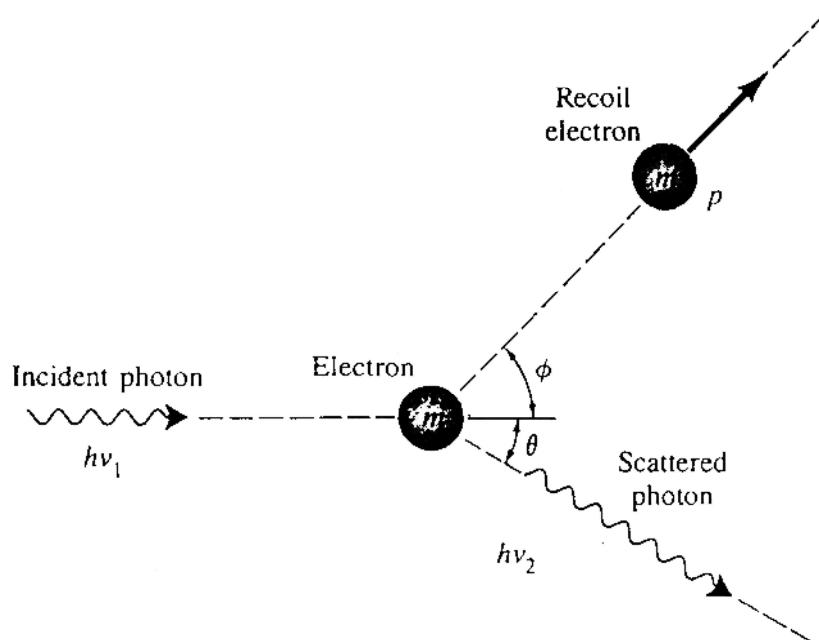
$$v = eV / h ,$$

and wavelength

$$ch / eV = \lambda$$

The confirmation that in X ray diffraction

$$ch / eV = 2d \sin \theta \qquad\qquad (31)$$

proves that the X-ray particle is a photon of electromagnetic energy hv.

The second experiment that proves that X-rays are photons is the Compton effect in which an X-ray particle collides with an electron.

If we assume that the X-ray particle is a photon $h\nu_1$, that imparts energy E to an electron, and becomes a photon $h\nu_2$, we have

$$\frac{h\nu_1}{c} = \frac{h\nu_2}{c} + p\cos\theta$$

$$\frac{h\nu_2}{c}\sin\varphi = p\sin\theta \, .$$

Thus,

$$(h\nu_1)^2 + (h\nu_2)^2 - 2h^2\nu_1\nu_2\cos\varphi = (pc)^2 = E^2 - (m_e c^2)^2 \, .$$

Since

$$h\nu_1 + m_e c^2 = h\nu_2 + E \, ,$$

we obtain

$$c^2 = \frac{\nu_1\nu_2}{\nu_1 - \nu_2}\frac{h}{m_e}(1 - \cos\varphi) \, . \qquad (32)$$

The confirmation of this formula in the Compton experiment, validates the assumption that the X-ray particle is a photon. That is, a particle of electromagnetic energy.

Since X-rays are photons, the uncertainty equals their wavelength. X-rays wavelength is determined in crystal diffraction.

7. Dispersion Relations

For a relativistic particle, the energy relation

$$E^2 = (mc^2)^2 = (m_0 c^2)^2 + (pc)^2 \qquad (33)$$

is equivalent to

$$m^2 c^4 = m^2(1 - \upsilon^2/c^2)c^4 + (pc)^2$$

or

$$p = m\upsilon \, . \qquad (34)$$

Denote

$$2\pi / \lambda = k .$$

Substituting in (34)

$$E = hv = mc^2 , \text{ and } p = h/(\lambda c / \upsilon) ,$$

we obtain the dispersion relation

$$h\upsilon / \lambda c = (hv / c^2)\upsilon$$

$$c 2\pi / \lambda = 2\pi v$$

$$ck = \omega .$$

For a non-relativistic particle, the energy equation

$$\frac{1}{2}m\upsilon^2 = E - V ,$$

is equivalent to

$$E = \frac{1}{2m}p^2 + V . \qquad (35)$$

Substituting in (35) $E = hv$, $p = h/(\lambda c / \upsilon)$, we obtain the dispersion relation

$$hv = \frac{1}{2m}\frac{h^2\upsilon^2}{\lambda^2 c^2} + V$$

$$\omega = \frac{\hbar}{2m}\frac{\upsilon^2}{c^2}k^2 + \frac{V}{\hbar} .$$

8. Strings and Uncertainty

de Broglie's association of virtual waves with a particle lead to the seeking of particle diffraction. But there are no matter waves, the relation breaks down at $\upsilon = 0$, and de Broglie later analysis promotes uncertainty over his virtual waves.

The Strings postulate reinstates the matter wave by having the particle itself vibrate in its uncertainty region. In that region, the energy and the momentum of the particle are unknown, and the trajectory of the particle cannot be determined.

The string postulate cannot be confirmed, or denied because no experiment may apply to the uncertainty region.

In particular, vibrations in a region where physics cannot be quantified, need not lead to any physical results, whatever the number of unseen space dimensions may be.

String theory aims to resolve singularities such as $1/x$, for $x \approx 0$. But in quantum physics this problem is already resolved by uncertainty: Away from the uncertainty region Δx, we compute $1/x$. In the uncertainty region, energy or momentum cannot be determined, the equation may not hold, and we do not apply it.

There is no way to replace the uncertainty inherent in the quantum theory with certainty, anymore than there is a way to replace the quantum hypothesis with wave theory. In fact, we may sum up the foundations of the quantum theory as the combination of the Planck-Einstein quantum hypothesis, and the de Broglie-Heisenberg uncertainty relations. Uncertainty is a fundamental property of the quantum theory.

Thus, it should not be surprising that the result of avoiding uncertainty, and reconstructing quantum theory without it, leads nowhere. The failure of this exercise in avoiding uncertainty, reaffirms how fundamental the Heisenberg Uncertainty relations are.

As Feynman's observed in [6], the string postulate may not be the only way to resolve singularities, and there are other possibilities for resolving the singularity. For instance, one such possibility is that the equation is wrong at the singularity.

References

1. de Broglie, Louis, *"The Beginnings of Wave Mechanics"* in *"Wave mechanics the first fifty years"* edited by Price, William; Chissick, Seymour; Ravensdale, Tom . Halsted Press (Wiley) 1973. p. 13.

2. Bass, Jean, *"Probability, PseudoProbability, Mean Values"* in *"Wave-Particle Duality"*, edited by Franco Selleri, Plenum1992, p. 5.

3. Rousseau, M., and Mathieu, J. P., "Problems in Optics", Pergamon Press, 1973, p. 246.

4. de Broglie, Louis, *"Heisenberg's Uncertainties and the Probabilistic Interpretation of Wave Mechanics"*, Kluwer, 1990.

5. Flint, H. T., *"Wave Mechanics"* Methuen, 1953, p. 66

6. Feynman, Richard, in *"Superstrings, a theory of everything"* edited by P.C.W. Davis, and J. Brown, Cambridge Univ. Press, 1988, pp. 192-210.