GR9677 Solutions

Detailed Solutions to the GRE Physics Exam

http://grephysics.yosunism.com

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December 6, 2005

Problem 1 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=1*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *RC Circuit*

One can immediately eliminate plots A, C, E, since one would expect an exponential decay behavior for current once the switch is flipped.

More rigorously, the initial circuit with the switch S connected to a has the following equation,

$$V - \dot{Q}r - Q/C = 0 \Rightarrow \frac{dQ}{dt} = \frac{1}{r}\left(V - Q/C\right) \quad (1)$$

Once integrated, the equation becomes,

$$Ve^{-\frac{t}{rC}} = V - Q/C \Rightarrow Q/C = V\left(1 - e^{-\frac{t}{rC}}\right) \quad (2)$$

The charge stored on the capacitor after it is fully charged is $Q = CV$ (at $t = 0$).

When the switch S is switched to b, the equation becomes,

$$Q/C = \dot{Q}R \Rightarrow Q(t) = Q_0 e^{-\frac{t}{RC}}, \quad (3)$$

where $Q_0 = CV$ from the initial connection.

Current is the *negative* time derivative of charge, and thus,

$$I(t) = -\dot{Q} = \frac{Q_0}{RC}Q(t) = \frac{V}{R}Q(t) \quad (4)$$

The initial current IS $I(0) = V/R$, and thus choice (B) is right.

Problem 2 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=2*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Faraday Law*

Recall Faraday's Law $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$. Dot both sides with the area $d\vec{A}$. Recalling Stokes' Theorem ($\int \nabla \times \vec{E} \cdot d\vec{A} = -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$), the left side can be converted to the potential, i.e., the emf $\mathcal{E} = -\int \vec{E} \cdot d\vec{l} = -\int \nabla \times \vec{E} \cdot d\vec{A}$.

Finally, from Ohm's Law $V - \mathcal{E} = IR$, one can obtain the current. (Note that $V = 5.0$ V is the voltage of the battery. The voltage induced acts to oppose this emf from the battery.)

The problem gives $\frac{dB}{dt} = 150T/s$. The area is just $0.1^2 m^2$. Thus, the induced emf is,

$$\mathcal{E} = \frac{dB}{dt}A = 150/100 = 1.5 \quad (5)$$

Thus, $V - \mathcal{E} = 3.5 = IR \Rightarrow I = 0.35A$, since $R = 10\Omega$.

Problem 3 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=3*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Potential*

Recall the elementary equations, $V = \int \vec{E} \cdot d\vec{l} = \int \frac{dq}{4\pi\epsilon_0 r}$.

$r = \sqrt{R^2 + x^2}$, and $dQ = Q$ thus $V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+x^2}}$, as in choice (B).

Problem 4 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=4*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Small Oscillations*

The potential is determined in the previous problem to be $V = \frac{Q}{4\pi\epsilon_0\sqrt{R^2+x^2}}$. The field is given by $\vec{E} = -\nabla V$. Before taking derivatives, one can simplify the potential since it is given that $R >> x$.

Binomial expand it ($(1+y)^n \approx 1+ny$, for y small) to get

$$V \approx \frac{Q}{4\pi\epsilon_0 R}(1+\frac{x^2}{2R^2}) \tag{6}$$

Taking the derivative, using the equations $\vec{E} = -\nabla V$, and $\vec{F} = q\vec{E}$, one gets,

$$-q\frac{Q}{4\pi\epsilon_0 R}\frac{x}{R^2} = F = m\ddot{x}. \tag{7}$$

Small oscillations have the same form as simple harmonic oscillations, i.e., $\ddot{x} = -\omega^2 x$. The angular frequency is $\omega = \sqrt{q\frac{Q}{4\pi\epsilon_0 R^3 m}}$, as in choice (A).

Problem 5 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=5*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Centripetal Force*

$\vec{F}_a ir$ acts in the direction as shown and the centripetal acceleration acts in the direction of $\vec{F}_A$. Centripetal acceleration is a net force, however, and thus,

$$\rightarrow_+ \sum F_x = 0 \quad = \quad -F_{air} + f_x \tag{8}$$

$$\uparrow_+ \sum F_y = -mv^2/r \quad = \quad f_y \tag{9}$$

f_x is in the positive direction and f_y is in the negative direction. Thus the force of the road is $\vec{F}_D$.

Problem 6 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=6*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Inclined Plane*

Set up the usual coordinate system with horizontal axis parallel to incline surface. The equations are, (since the mass slides down at constant speed),

$$\sum F_x \quad = \quad 0 = f - mg\sin\theta \tag{10}$$

$$\sum F_y \quad = \quad 0 = N - mg\cos\theta \tag{11}$$

Friction is given by $f = \mu N = \mu mg\cos\theta$, where the normal force N is determined from the F_y equation. For constant velocity one also has, $f = mg\sin\theta = \mu mg\cos\theta \Rightarrow \mu = \tan\theta$

To find the work done by friction, one calculates $W = fL$, where $L\sin\theta = h$. Thus $W = \tan\theta mg\cos\theta\frac{h}{\sin\theta} = mgh$, as in the almost-too-trivial, but right, choice (B).

Problem 7 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=7*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Elastic Collisions*

One determines the velocity of impact of the ball from conservation of energy,

$$mgh = \frac{1}{2}mv_0^2 \Rightarrow v_0^2 = 2gh \tag{12}$$

Conservation of momentum gives,

$$v_0 = -v_1 + 2v_2 \tag{13}$$

Conservation of *kinetic* energy gives,

$$mv_0^2 = mv_1^2 + 2v_2^2 \tag{14}$$

Plug in the momentum and kinetic energy conservation equations to solve for v_1 and v_2 in terms of v_0 to get

$$v_1 = -v_0/3 \tag{15}$$

$$v_2 = 2v_0/3 \tag{16}$$

Write yet another conservation of energy equation for the *final* energy,

$$\frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2 = mgh' + \frac{1}{2}2mv_2^2, \tag{17}$$

where the condition that the mass $2m$ slides on a *frictionless* plane is used.

Thus, $\frac{1}{2}mv_1^2 = \frac{v0^2}{18} = mgh' \Rightarrow h' = \frac{h}{9}$, where the previous result $v_1 = -v_0/3$ and $v_0^2 = 2gh$ is used.

Problem 8 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=8*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Damped Oscillations*

One should remember that damped oscillations have decreasing amplitude according to an exponential envelope. As the amplitude shrinks, the period increases. The additional force instated in the problem is equivalent to damping, and thus the period increases, as in choice (A).

Problem 9 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=9*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Rydberg Energy*

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_0^2}\right)$. Given the information that the Lyman Series is $n_f = 1$, and the Balmer series is $n_f = 2$, one forms the ratio $\lambda_L/\lambda_B = 0.25$ (taking $n_i = \infty$). This is closest to choice (A). (Recall that ETS wants the answer that best fits.)

Problem 10 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=10*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Particle Physics*

Recall that in gamma-ray production, the excited nucleus jumps to a lower level and emits a photon γ. In internal conversion, however, an orbital electron is absorbed and ejected along with an *X-ray*.

Problem 11 $http://grephysics.net/disp.php?yload=prob\&serial=2\&prob=11$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Stern-Gerlach*

Recall the Stern-Gerlach experiment, where (in its original set-up) a beam of *neutral* silver atoms are sent through an inhomogeneous magnetic field. The beam's split into two—classically, from the Lorentz force, one wouldn't expect anything to happen since all the atoms are neutral, but if one accounts for the Larmor precession, one would expect the beam to be deflected into a smear. Instead, however, the beam deflects into $2s + 1$ beams, and thus this supports the idea that electrons are of spin-1/2. (Ag has one unpaired electron in its p orbital.)

With a beam of hydrogen atoms, one should also get a split into two, since $s = 1/2$ from the electron.

Problem 12 $http://grephysics.net/disp.php?yload=prob\&serial=2\&prob=12$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Positronium*

The positronium atom involves a positron-electron combination instead of the usual proton-electron combo for the H atom. Charge remains the same, and thus one can approximate its eigenvalue by changing the mass of the Rydberg energy (recall that the ground state of the Hydrogen atom is 1 Rydberg).

Recall the reduced mass $\mu = \frac{m_1 m_2}{m_1+m_2}$, where for identical masses, one obtains $\mu = m/2$. The Rydberg in the regular Hydrogen energy eigenvalue formula $E = R\left(1/n_f^2 - 1/n_i^2\right)$ is proportional to μ. Substitute in the new value of the reduced mass to get $E \approx R/2$. $R = -13.6$ eV, and thus $E \approx -6.8$ eV.

Problem 13 $http://grephysics.net/disp.php?yload=prob\&serial=2\&prob=13$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Heat*

Given $P = 100$ W and $V = 1L = 1m^3 = 1kg$ for water, one can chunk out the specific heat equation for heat, $Q = mc\Delta T = Pt \Rightarrow 4200(1^\circ) = 100t \Rightarrow t \approx 40s$, as in choice (B).

Problem 14 $http://grephysics.net/disp.php?yload=prob\&serial=2\&prob=14$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Heat*

The final temperature is $50^\circ C$. The heat exchanged from the hot block to the cool block is $Q = mc\Delta T = 5kcal$, as in choice (D).

Problem 15 $http://grephysics.net/disp.php?yload=prob\&serial=2\&prob=15$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Phase Diagram*

Recall that for an ideal gas $U = C_v \Delta T$ and $PV = nRT$. Don't forget the first law of thermodynamics, $Q = W + U$.

For $A \rightarrow B$, $U = 0$, since the temperature is constant. Thus, $Q = W = RT_H \ln V_2/V_1$.

For $B \rightarrow C$, $W = P_2(V_1 - V_2) = R(T_c - T_h)$. $U = C_v(T_c - T_h)$, and thus $Q = W + U = C_v(T_c - T_h) - R(T_h - T_c)$.

For $C \rightarrow A$, $W = 0$, $U = C_v(T_h - T_c)$, thus $Q = U = C_v(T_h - T_c)$.

Add up all the Q's from above, cancel the C_v term, to get $Q_t ot = RT_h \ln(V_2/V_1) - R(T_h - T_c)$, as in choice (E).s

Problem 16 ________________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=16*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Mean Free Path*

Air is obviously less dense than the atomic radius 10^{-10}, thus choices (C), (D), and (E) are out. Air is not dilute enough that the distance between particles is actually within human visible range, as in (A)! Thus, the answer must be (B). (Note how this problem exemplifies the usefulness of common sense.)

Problem 17 ________________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=17*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Probability*

The careless error here would be to just directly square the grids. When one remembers the significance of the meaning of the probability $P = \int |\langle\psi|\psi\rangle|^2 dV$, one finds that one must square the wave function, and not the grids.

The total probability is,

$$\int_0^6 |\psi|^2 = 1 + 1 + 4 + 9 + 1 + 0 = 16 \tag{18}$$

The un-normalized probability from $x = 2$ to $x = 4$ is,

$$\int_2^4 |\psi|^2 = 4 + 9 = 13 \tag{19}$$

The normalized probability is thus 13/16, as in choice (E).

Problem 18 ________________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=18*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Scattering*

This is a conceptual scattering question. No calculations needed.

(A) A particle incident from the left would have an oscillating wave function until it meets the barrier... not the other way around.

(B) For $E < V_0$, It is true that the barrier would decrease the amplitude of the wave function, however, when it emerges, the tunneled part of it should have an even smaller amplitude. (This graph would be good for $E > V_0$, however.)

(C) This is the only graph that shows the wanted characteristics: oscillating wave before incidence, decay while in barrier, and tunneled-decrease amplitude when exit.

(D) A typical particle is probably least likely to be found inside the barrier, so this is the least likely choice.

(E) This wave function shows no change, when the potential barrier demands a change!

Problem 19 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=19*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Coulomb's Law*

The particle obeys a Coulomb's Law potential, $V = \frac{kZ_1Z_2q^2}{r}$. In this case particle 1 is a Helium atom, which has charge $Z_1 = 2$, while particle 2 is silver, with $Z_2 = 50$. Thus,

$$V = \frac{100kq^2}{r}. \tag{20}$$

Conservation of energy requires that when the incident particle is at its closest approach, $5MeV = \frac{100kq^2}{r}$. Recall that $k = 9E9$, $q = 1.602E - 19$, convert everything to SI to get $r \approx 2.9E - 14$.

Problem 20 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=20*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation of Momentum*

The Helium atom (m) makes an elastic collision, and thus the kinetic energy before and after is conserved.

$$\frac{1}{2}mv^2 = \frac{1}{2}m(0.6v)^2 + \frac{1}{2}MV'^2 \Rightarrow 0.64mv^2 = MV'^2 \tag{21}$$

Conservation of momentum requires that,

$$mv = -.6mv + MV' \Rightarrow V' = 1.6mv/M \tag{22}$$

From kinetic energy conservation, $0.64mv^2 = MV'^2 \Rightarrow 0.64mv^2 = (1.6mv)^2/M \Rightarrow 0.64 = 1.6^2m/M \Rightarrow M = 1.6^2m/0.64 = 4m$, but since $m = 4u$, $M = 16u$, for O_2, as in choice (D).

Problem 21 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=21*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Moment of Inertia*

To solve this problem, one should remember the parallel axis equation to calculate the moment of inertia about one end of the hoop: $I = I_{cm} + md^2 = mR^2 + md^2 = 2mR^2$, where d is the distance from the pivot point to the center of mass, which in this problem, is just equal to R. (In the last equality, note that the moment of inertia of a hoop of radius R and mass m about its center of mass is just $I_{cm} = mR^2$.)

The problem gives the period of a physical pendulum as $T = 2\pi\sqrt{I/(mgd)}$. Thus, plugging in the above result for the moment of inertia, one has, $T = 2\pi\sqrt{2mR^2/(mgR)} = 2\pi\sqrt{2R/(g)} \approx 2 * 3\sqrt{2 * 0.2/(10)} = 12/10 = 1.2s$, which is closest to choice (C). (Since π was rounded to 3, the period should be slightly longer than 1.2s.)

Problem 22 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=22*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Geometry*

The harder part of this problem involves determining the radius of Mars. It's an approximate geometry problem. The problem gives a vertical drop of 2m for every 2600m tangent to the surface. The tangent to the surface is approximately one leg of a triangle whose hypotenuse is the radius of Mars, since the radius is much larger than the tangent distance. The other leg of the right triangle is just $r-2$, where r is the radius of Mars. In equation form, what was just said becomes $(r-2)^2+3600^2 = r^2$. The square terms cancel out, and dropping out the 4, one has $r \approx 3600^2/2 \approx 8E6m$.

(The above deduction was due to Ayanangsha Sen.)

The easier part comes in the final half of the problem: applying the centripetal force to the force of gravity. $mv^2/r = mg \Rightarrow v = \sqrt{2gr} \approx \sqrt{20 * 8E6} = \sqrt{16E7} \approx 4000m/s$, which is closest to $3.6km/s$, as in choice (C).

Problem 23 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=23*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Stability of Orbits*

The gravitational force suspect to a bit of perturbation is given as $\vec{F}_{12} = \hat{r}_{12} G m_1 m_2 / r_{12}^{2+\epsilon}$.

One can narrow down most choices by recalling some basic facts from central force theory:

(A) No mention is made of frictional effects, and thus energy should be conserved.

(B) Angular momentum is always conserved since the net torque is 0 (to wit: the force and moment arm are parallel).

(C) This is just Kepler's Third Law applied to this force. (Recall the following bromide: The square of the period is equal to the cube of the radius—for the inverse square law force. For a perturbed force, the bromide becomes: The square of the period is equal to the $3+\epsilon$ power of the radius.)

(D) Recall Bertrand's Theorem from Goldstein. Stable non-circular orbits can *only* occur for the simple harmonic potential and the inverse-square law force. This is of neither form, and thus this choice is FALSE.

(E) Circular orbits exist for basically all potentials. A stationary orbit exists if and only if the following conditions are satisfied: $V' = 0 \quad V'' > 0$. Recall that the potential is related to the force by $-V' = F \Rightarrow V = -\int F dx$. Use $V \propto 1/r^n$, and recalling the extra term added to the effective potential to be $L^2/(2mr^2)$, one chunks out the derivatives to get the condition that $n < 2$, as a potential exponent, ($n < 3$, as a force exponent) for stable orbit. One can remember this result or re-derive it whenever necessary. For $n < 3$, (the power exponent of the force equation), a stable circular orbit exists. Since ϵ is presumably less than 1, the planet does, indeed, move in a stationary circular orbit about the sun.

Problem 24 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=24*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Conductors*

This problem involves applying Coulomb's Law $F \propto q_A q_B / r_{AB}^2$ to conductors. The charge travels from conductor to conductor and equilibriates instantaneously due to the requirement that two touching conductors must be at an equipotential. This means that if conductors 1 and 2 touch then their potentials are related by $V_1 = V_2$. Because the problem involves spherical conductors, the potential has the form $V \propto q_A q_B / r_{AB}$.

The initial force between the two conductors is F, where $q_A = q_B = Q$.

After C is touched to A, the charge becomes $q_A = Q/2 = q_C$, since each conductor shares the same charge out of a total of Q (to wit: each has half of the total charge).

When C is touched to B, the charge becomes $q_C = 3/4Q = q_B$, since each conductor shares the same charge out of a total of $Q + Q/2$ (to wit: $\frac{1}{2}3/2Q = 3/4Q$ for each conductor).

When C is removed, one calculates the force from Coulomb's law and the final charges on A and B determined above to be, $F = 3/8Q^2/r_{AB}^2 = 3/8F$, as in choice (D).

Problem 25 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=25*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

***Electromagnetism** → Capacitors*

Recall the following truths (held to be self-evident?) on the subject of capacitors: 1. series capacitors have equal charge (Equivalent capacitance of two capacitors is $1/C_{eq} = 1/C_1 + 1/C_2$); 2. parallel capacitance have equal voltage ($C_{eq} = C_1 + C_2$); 3. $Q = CV$; 4. $U = \frac{1}{2}CV^2$.

(A) Initially, before the switch is closed, only C_1 has a voltage across it, and hence it is charged. $Q_0 = CV$. But, afterwards, since the voltage stays the same, one has $Q_1 = Q_2 = CV$; hence, $Q_0 = \frac{1}{2}(Q_1 + Q_2)$.

(B) $V_1 = V_2 \Rightarrow Q_1/C_1 = Q_2/C_2$. Since $C_1 = C_2$, one has $Q_1 = Q_2$. This is true.

(C) By definition of circuit elements in parallel, one has each capacitor at the same potential. This is trivially true. $V_1 = V_2 = V$

(D) Since one determined from (C) that the capacitors are at the same voltage, then because they have the same capacitance, they have the same energy as per $U = \frac{1}{2}CV^2$. True.

(E) This is false, since $U_0 = \frac{1}{2}CV^2$, initially. In the final state, *each* capacitor has energy $\frac{1}{2}CV^2$. The sum of energies is thus $2U_0$.

Problem 26 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=26*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

***Electromagnetism** → Resonance Frequency*

One wants to tune one's radio to the resonance frequency (a.k.a. the frequency at which impedance is matched). The resonance frequency of an LRC circuit is given by $\omega^2 = 1/LC$, where the quantities involved are angular frequency, inductance, and capacitance. Solving for C, one has $C = 1/(L\omega^2) \approx 1/(2E - 6 * 36 * 100E6) = 1/(7.2E - 11) \approx 0.1E - 11 = 1E - 12$. This is choice (C). The hardest part of his problem, of course, is doing the math without a calculator. Easy.

Problem 27 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=27*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

***Lab Methods** → Log Graphs*

Log graphs are good for exponential-related phenomenon. Thus (A), (C), and (E) are appropriate, thus eliminated. The stopping potential has a linear relation to the frequency, and thus choice (B) is eliminated. The remaining choice is (D).

Problem 28 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=28*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

***Lab Methods** → Oscilloscope*

A superposition of two oscillations has the form $\sin\omega_1 t + \sin\omega_2 t = 2\cos(\frac{\omega_1 - \omega_2}{2})\sin(\frac{\omega_1 + \omega_2}{2})$. This implies that the cosine term is the amplitude of the combined wave.

Similarly, one can see one the lower frequency as the contribution towards the bigger envelope-like wave and the higher-frequency as the zig-zag-gish motion along the envelope.

One oscillation must have a high frequency and the other has a relatively lower frequency. Only choices (D), (A), and (B) show this trait. The high frequency oscillation should have a smaller amplitude than the lower frequency oscillation. Only choices (A) and (D) show this trait. Finally, the amplitude of the lower frequency wave forms the envelope, and the amplitude from that is only about 1 cm; on a 2V scale, this is about 2V—which is closest to choice (D).

Problem 29 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=29*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Methods → *Dimensional Analysis*

The current author is fortunate enough to have taken a String Theory course as an undergraduate, and thus know by heart that the Planck length is $\sqrt{G\hbar/c^3}$. However, the problem can also be solved via dimensional analysis

(A) $G\hbar c$ has units of $\left(m^3/(kgs^2)\right)(kgm/s^2ms)(m/s) = m^5/(s^3)$, which doesn't have the units of m.

(B) $\left(m^3/(kgs^2)\right)(kgm/s^2ms)^2(m/s)^3 = m^{10}kg/s^7$, which doesn't have the units of m.

(C) $\left(m^3/(kgs^2)\right)^2(kgm/s^2ms)(m/s) = m^9/(kgs)$, which doesn't have the units of m.

(D) $\left(m^3/(kgs^2)\right)^{1/2}(kgm/s^2ms)^2(m/s) = \sqrt{kg}m^{4.5}/s^3$, which doesn't have the units of m.

(E) This is the last one. Take it!

Problem 30 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=30*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Fluid Mechanics*

Equipotential leads to equipressure in a fluid. Thus, take the pressure at the base of the dark fluid and set it equal to the pressure (of the lighter-colored fluid) at a horizontal-line across on the right-hand side of the U:

$P_{dark} = \rho_4 g(5) = P_{light} = \rho_1 g(h_2 - (h_1 - 5)) = \rho_1 g(h_2 - h_1 + 5)$. The initial total height of the columns is 40, thus after the darker liquid is added, the total height is 45. Plug $h_1 + h_2 = 45$ into the equation above to get $h_1 = 15$, $h_2 = 30$, and therefore $h_2/h_1 = 2/1$, as in choice (C).

(Ah, one should remember that the fluid pressure at a point is due to all the water on top of it, thus $P = \rho gh$, where h is the height of the water on top of the point.)

Problem 31 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=31*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Frictional Force*

(A) A falling object experiencing friction falls faster and faster until it reaches a terminal speed. Its kinetic energy increases proportional to the square of the velocity and approaches a asymptotic value.

(B) The kinetic energy increases to a maximum, but it does not decrease to 0. See (A).

(C) The maximal speed is the terminal speed.

(D) One has the equation $m\ddot{y} + b\dot{y} + mg = 0 \Rightarrow m\dot{v} + bv + mg = 0$. Without having to solve for v, one can tell by inspection that $v(t)$ will depend on both b and m.

(E) See (D). This is the remaining choice, and it's right.

Problem 32 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=32*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Moment of Inertia*

The inertia through the point A is $I_A = 3mr^2$. From geometry, one deduces that the distance between each mass and the centerpoint A is $r\cos(30°) = l/2 \Rightarrow r = l/\sqrt{3}$. The moment of inertia about A is thus $I_A = ml^2$

The inertia about point B can be obtained from the parallel axis theorem ($I_{displaced} = I_{cm} + \sum_i m_i d^2$, where d is the displaced distance from the center of mass). Because $d = l/\sqrt{3}$, one has $I_B = I_A + 3md^2 = 2I_A$. Since the angular velocity is the same for both kinetic energies, recalling the relation for kinetic energy $K_i = I_i\omega_i^2$, one has $K_B/K_A = I_B/I_A = 2$, as in choice (B).

Problem 33 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=33*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Probability*

Recall that $P(apple) = \int |\langle\psi_{apple}|\psi_{fruit}\rangle|^2 dx$

Given the wave function in terms of the spherical harmonic eigenfunctions, one has it totally easy. One has $\langle\psi| = (3Y_5^1 + 2Y_5^{-1})/\sqrt{38}$. Ketting the bra, one has, $P = (9+4)/38$, where one recalls the orthonormality of the spherical harmonic eigenfunctions. This is choice (C).

Problem 34 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=34*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Particle Physics*

Regularly, electrons are emitted in any direction. Thus, there is *infinite* symmetry. In the case of a magnetic field, electrons are more likely to be emitted in a direction opposite to the spin direction of the decaying atom. Place the atom in an x-y plane, with its spin-direction pointing along the z-axis. If the electron is mostly emitted in the -z axis, then reflection symmetry is violated since it's not (mostly) emitted in the +z axis, i.e., not mirrored across the x-y plane. Choice (D). (This is due to Joe Bradley.)

Problem 35 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=35*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Identical Particles*

Because of the antisymmetric interchange of identical particles, one gets $\psi = 0$ if two fermions are in the same state. This is basically the foundation behind the familiar Pauli exclusion principle.

Problem 36 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=36*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Conservation of Energy*

The rest mass for each mass is 4kg. They collide head-on with identical speeds pointing in opposite directions. This implies that the composite mass is at rest. Thus, recalling that the total energy is given by $E = \gamma mc^2$ and that the rest mass is given by $E = mc^2$, one has $2\gamma mc^2 = Mc^2$, where M is the composite mass.

The particle travels at $v = 3c/5$, which yields $\gamma = 5/4$. Plug this in to get $M = 10/4 \times 4 = 10kg$.

Problem 37 ________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=37*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Addition of Velocities*

Recall that $u' = \frac{v+u}{1+vu/c^2}$. The problem gives $u = 0.6c$, $v = 0.3c$. Thus, $u' = \frac{0.9c}{1+0.18}$, which is slightly less than choice (E). Choose choice (D).

If one forgets the addition of velocity formula, one can always derive it from taking derivatives of the Lorentz Transformations, which are easier to remember, with $x' = \gamma(x + vt)$ and $t' = \gamma(t + vx/c^2)$.

Problem 38 ________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=38*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Energy*

The problem gives $\gamma mc^2 = 10$ and $\gamma mv = 5$. Divide the two to get $v/c^2 = 0.5 \Rightarrow v = c/2$, as in choice (D). The hardest part of the problem is remembering the definition of relativistic energy in terms of just the rest mass and γ.

Problem 39 ________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=39*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Ionization Potential*

Atoms with full shells have high ionization potentials—they would hardly want to lose an electron, and thus it would take a great amount of energy to ionize them. Atoms with close-to-full-shells have similarly high potentials, as compared to atoms that are in the middle of the spectrum, take $Cs^5 5$, for example.

(A) He has a full orbital, and thus its ionization potential must be high.
(B) N has a close-to-full orbital.
(C) O has a close-to-full orbital.
(D) Ar is a nobel gas, and thus its orbital is full.
(E) This is it. Cs has the lowest IP of all (of the above).

Problem 40 ________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=40*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Bohr Theory*

It's amazing how far one can get with the Bohr formula.

To start with, one should calculate the ground-state energy of the singly ionized Helium (i.e., the ionization energy). $E_1 = Z^2 E_{H1} = 4 \times 13.6eV$, since Helium has 2 protons. (The general formula is $E_n = Z^2/n^2 E_1$.)

The Bohr formula gives $E = E_1 \left(1/n_f^2 - 1/n_i^2\right) = E_1(1/n_f^2 - 1/16)$, since $n_i^2 = 4^2 = 16$.

$E = hc/\lambda \approx 1.24E - 6/4.7E - 7$ gives $E \approx 2.5$ eV.

The only unknown expression above is n_f. Plugging everything in and solving for that, $n_f^{-2} \approx 8^{-1} \Rightarrow n_f \approx 3$. This yields choice (A). One can check via $E_f = E_1/n_f^2 4 \times 13.6/9 \approx 6$, which verifies (A).

Problem 41 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=41*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Atomic → *Spectroscopic Notations*

Recall the selection rules $\Delta m = 0, \pm 1$ and $\Delta l = \pm 1$ for the electric dipole approximation in time-dependent perturbation theory.

(A) This is allowed since $\Delta l = -1$ and $\Delta m = 1$
(B) This is *not* allowed since $\Delta l = 0$, which goes against the condition that $\Delta l = \pm 1$.
(C) $l = 1$ for p orbitals. Recall $(s, p, d, f) = (0, 1, 2, 3)$
(D) An electron has $s = 1/2$, thus one can't have $j = l$ or $s = 3/2$.
(E) One does not know this for sure. Choice (A) is the best choice.

Problem 42 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=42*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Photoelectric Effect*

Recall the photoelectric equation relating the incident electromagnetic wave to the kinetic energy and the work function $hc/\lambda = K + \phi \; approx 12E - 7/500E - 9 = K + 2.28 \Rightarrow K = 12E - 7/5E - 7 - 2.28 \approx 0.2eV$, as in choice (B).

Problem 43 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=43*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Stokes Theorem*

Recall Stokes' Theorem $\oint \nabla \times u d\vec{a} = \int \vec{u} \cdot d\vec{l}$. The left side of the equality is easier to evaluate, so evaluating that, one has $\nabla \times u = 2$. The area is πR^2, and thus $\int \vec{u} \cdot d\vec{l} = 2\pi R^2$.

Problem 44 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=44*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Chain Rule*

Recall that $a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$. $\frac{dv}{dx} = -n\beta x^{-n-1} \Rightarrow v\frac{dv}{dx} = -n\beta^2 x^{-2n-1}$, as in choice (A).

Problem 45 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=45*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Lab Methods → *High-pass filter*

Recall the impedance formulae for capacitors $X_c = \frac{1}{\omega C}$ and inductors $X_L = \omega L$. The complex impedance is $Z = -iX_C + iX_L + R$, and the ac-version of Ohm's Law becomes: $V = IZ$.

For choice (E), one has $Z = R + iX_C \Rightarrow V_{in} = I\left(R + i(X_L - X_C)\right) \Rightarrow I = \frac{V_{in}(R+iX_C)}{R^2+X_C^2}$, where in the last step, one multiplies top and bottom by the complex conjugate of the denominator impedance Z. The voltage across the resistor is the voltage from ground, thus $V_{out} = IR = \frac{V_{in}(R+iX_C)R}{R^2+X_C^2}$.

For high frequencies, one has $\omega \to \infty \Rightarrow X_C \to 0 \Rightarrow V_{out} = V_{in}R^2/R^2$.

For low frequencies, one has $\omega \to 0 \Rightarrow X_C >> 1 \Rightarrow V_{out} = \frac{V_{in}R}{X_C} \to 0$.

Circuit (E) meets the given conditions.

(Incidentally, choice (D) is a low-pass filter giving $V_{out} \to 0$ for high frequencies.)

(For more on this, check out Horowitz' *The Art of Electronics*.)

Problem 46 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=46*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Faraday Law*

Recall Faraday Law, $\epsilon = -\frac{d\Phi}{dt}$, where $\Phi = B \cdot dA$. Since the magnetic field is constant, the equation simplifies to $\epsilon = -B \cdot \frac{dA}{dt}$ for this case.

$B \cdot A = B\cos(\omega t)\pi r^2$, and thus $d\Phi/dt = -\omega\sin(\omega t)\pi r^2 = -\epsilon = -\epsilon_0\sin(\omega t)$. Solving for angular momentum, one has $\omega = \epsilon_0/(B\pi R^2)$.

Alternatively, one has $A = \pi r r(t) \Rightarrow \dot{A} = \pi r\dot{r} = \pi r v$. Since $v = \omega r\sin(\omega t)$, one has $A = \pi r\omega r\sin(\omega t)$. Plug it into Faraday Law and solve for angular velocity.

Problem 47 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=47*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Faraday Law*

Recall Faraday's Law, $\epsilon = \vec{E} \cdot d\vec{l} = -d\Phi/dt$, where $\Phi = \vec{B} \cdot d\vec{A}$. In words, this means that a changing flux (either a varying field or radius) induces a voltage.

The field is given as just B. The area of the loop is just πR^2, i.e., the cross-sectional area of the cylinder. As the cylinder is spun around, its flux changes at the rate of N rps. The change in flux is thus $NB\pi R^2$, and this is the magnitude of the potential difference in choice (C).

(Also, one can drop out the other choices from units. And, since the cylinder is moving in a magnetic field, the non-zero flux demands a voltage, so (A) can't be it.)

Problem 48 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=48*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Half Life*

The halflife of the mesons is given. Since only half of the mesons reach point B 15 meters away, one presume that it takes 1 halflife of proper time to get there.

The proper time is $t_0 = 2.5E - 8$. The length $L = 15 = vt$ is in the lab frame, and since the time dilation equation gives $t = t_0\gamma$, one has $L = 15 = vt_0\gamma = vt_0/\sqrt{1-(v/c)^2} \Rightarrow L/t_0 = \frac{v}{\sqrt{1-(v/c)^2}}$.

Now, the gory arithmetics. No calculators allowed; 12 years of American public school mathematics wasted! $15/2.5E - 8 = 15/25E - 9 = 3/5E9 = 6E8 = \kappa = \frac{v}{1-(v/c)^2} = \frac{vc}{\sqrt{c^2-v^2}}$. Multiply things out to get $\kappa^2(c^2 - v^2) = v^2c^2 \Rightarrow \kappa^2c^2 = v^2(c^2 + \kappa^2)$. Plug in numbers to get $v = \frac{6E8c}{\sqrt{9E16+36E16}} = \frac{6E8c}{\sqrt{45E16}} = \frac{6c}{3\sqrt{5}}$, which is choice (C). Whew!

Problem 49 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=49*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Relativistic Fields*

For motion in the x direction, one has the following equations for the E and B fields,

$$E_x = E'_x \tag{23}$$
$$B_x = B'_x \tag{24}$$
$$E_y = \gamma(E'_y - vB'_z) \tag{25}$$
$$B_y = \gamma(B'_y - v/c^2 E'_z) \tag{26}$$
$$E_z = \gamma(E'_z - vB'_y) \tag{27}$$
$$B_z = \gamma(B'_z - v/c^2 E'_y) \tag{28}$$

Since $E'_z = \sigma/(2\epsilon_0)$ (with all other primed components 0), the transformed field is just $E_z = \gamma E'_z$, as in choice (C).

(Recall that $\gamma = 1/\sqrt{1-\beta^2}$, where $\beta = v/c$)

If one forgets the Lorentz-transformed fields, one can also quickly derive the answer for this case. Since the transformed charge density is Lorentz contracted in one of its area dimensions, one has $\sigma = \gamma\sigma'$. One can tell by symmetry of the surface that the other field components cancel, and one again arrives at the result for E_z as above.

Problem 50 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=50*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Spacetime Interval*

The spacetime interval is defined by the metric that negates spatial and time variables as $dS^2 = (cdt)^2 - (dx)^2$. dS is invariant. One has thus $dS^2 = dS'^2 \Rightarrow (3c)^2 = (5c)^2 - (ct)^2 \Rightarrow ct = 4c$ minutes, as in choice (C).

Problem 51 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=51*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Infinite Well*

The even wave functions always have nodes in the middle, and thus the probability density for states of even n vanish. (One can deduce this by fitting curves inside a box. The first state has no nodes in the middle, but a node at each end of the well. The second state has one node in the middle. The third state has two nodes, neither of which are in the middle. The fourth state has three nodes, one of which is in the middle.)

Problem 52 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=52*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Spherical Harmonics*

Y_l^m is the convention used for a spherical harmonic of eigenvalue m, l. The only spherical harmonic proportional to $\sin\theta$ is $Y_1^{\pm 1}$. Recalling that the eigen-equation, $L_z\psi = m\hbar\psi$, one deduces that since $m = \pm 1$, the eigenvalues are $\pm\hbar$.

(Open call for a better solution: Is there a method that does not require memorizing the first few spherical harmonics?)

Reproduced for the reader's pleasure and convenience, (proportional values of the first few SH's:

$$Y_0^0 = const \quad (29)$$
$$Y_1^0 = \cos\theta \quad (30)$$
$$Y_1^{\pm 1} = \sin\theta e^{\pm i\phi} \quad (31)$$

Problem 53 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=53*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Particle Physics*

One can ignore baryon numbers and lepton numbers and all that and just deal with spin conservation. For the positronium-electron spin singlet state, one has, initially, $s_i = 0$. The decay *must* conserve spin. Thus, one must have the final spin as $s_f = 0$. Since a photon is its own antiparticle (and antiparticles have the negation of the usual particle's quantum number), the photon has spin $s_p = 1$ and the antiphoton (just another photon) has spin $s_{\bar{p}} = -1$. Thus, two photons are emitted to conserve spin.

(Wheee... can one get more ad hoc than the Standard Model?)

Problem 54 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=54*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Field Trajectory*

The problem gives equi-amplitude, thus the field becomes $\vec{E} = Ee^{i(kz-\omega t)}\hat{x} + Ee^{i(kz-\omega t+\pi)}\hat{y}$. Taking the real part, (applying Euler's Theorem, to wit: $e^{i\theta} = \cos\theta + i\sin\theta$) one has $\vec{E} = E\cos(kz - \omega t)\hat{x} + E\cos(kz - \omega t + \pi)\hat{y}$. Apply the trig identity $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$ to make the field argument equi-phase, $\vec{E} = E\cos(kz - \omega t)\hat{x} - E\cos(kz - \omega t)\hat{y}$.

Looking down from the z-axis, one has $z = 0 \Rightarrow \vec{E} = E\cos(\omega t)\hat{x} - E\cos(\omega t)\hat{y}$.

Make a table of a few values of t and E,

$$\omega t \Rightarrow (\cos(\omega t), -\cos(\omega t)) \quad (32)$$
$$0 \Rightarrow (1, -1) \quad (33)$$
$$\pi/6 \Rightarrow \frac{1}{2}(\sqrt{3}, -\sqrt{3}) \quad (34)$$
$$\pi/4 \Rightarrow \frac{1}{2}(\sqrt{2}, -\sqrt{2}), \quad (35)$$

and one deduces that the points plot out a diagonal line at 135° to the x-axis, as in choice (B).

Problem 55 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=55*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Polarizations*

After the wave has been de-coupled into separate directions, the intensity adds separately. That is, the intensity of the wave split by the x-polarizer is $I_1 = |E_1|^2$, while that of the wave split by the y-polarizer is $I_2 = |E_2|^2$. Add the two intensities to get choice (A).

Problem 56 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=56*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Total Internal Reflections*

Total internal reflection is when one has a beam of light having *all* of the incident wave reflected. Going through a bit of formalism in electromagnetism one can derive Snell's Law for Total Internal Reflection,

$$n_{inside} \sin\theta = n_{outside}, \tag{36}$$

where $n_{inside} = 1.33$, and one assumes that the surface has $n_{outside} = 1$ for air.

One must solve the equation $\theta = \sin^{-1}(1/1.33)$. One can immediately throw out choices (A) and (E). From the unit circle, one recalls that $\sin(30^{\text{deg}}) = 1/2$ and $\sin(60^{\text{deg}}) = 1.7/2 = 0.85$. Since $1/1.33 \approx 0.7$, one deduces that the angle must be choice (C).

Problem 57 ________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=57*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Diffractions*

The single slit diffraction formula is $d \sin\theta = \lambda m$, where one has integer m for maxima and half-integers for minima. (Opposite to single-slit interference.)

Given $m = 1$, $\theta = 4E-3rad$, $\lambda = 400E-9m$, and making the approximation $\sin\theta \approx \theta$ for small angles, one has the following equation for d,

$d \approx \frac{\lambda m}{\theta} = \frac{4E-7}{4E-3} = 1E-4$, as in choice (C).

Problem 58 ________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=58*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Lensmaker Equations*

Although this problem mentions lasers, no knowledge of quantum mechanics or even lasers is required. Instead, the problem can be solved as a simple geometric optics problem using the lensmaker's equation, $1/d_i + 1/d_o = 1/f$, relating the distances of the object, image, and the focus.

Since there are two convex lenses, one can treat the set-up as a telescope. The lens closest to the laser is the objective (with focus f_o) and the one closest to the bigger-radius well-collimated beam is the eyepiece f_e.

The laser-light comes in from $d_{o1} = \infty$, and thus one has $d_{i1} = f_o$, i.e., the image forms at the focal point.

Using the telescope equation, one has $M = f_o/f_e = 10 \Rightarrow f_e = f_o/10 = 15cm$, since one wants a final magnification of 10 (to wit: input beam is 1mm, output beam is 10mm). This narrows down the choices to just (D) and (E).

Since one wants the distance of the image formed by the second lens to occur a good far distance away, one has $d_{i2} = \infty$, and thus $f_e = d_{o2} = 15cm$, as in choice (D).

Problem 59 ________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=59*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Laser*

Not much understanding of lasers is required to solve this one; the basic idea of the problem tests the relation between photon energy and energy from the laser. Recalling the equation $E =$

$hf = hc/\lambda \approx 12E-7/600E-9 = 2eV$, equates that to the energy (in eV) calculated from $Pt = 10E3 \times 1E-15/1.602E-19$. The answer comes out to choice (B).

Problem 60 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=60*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomena → *Light Doppler Shift*

One can derive the Doppler Shift for light as follows:

For source/observer moving towards each other, one has the wavelength emitted from the source decreasing, thus $\lambda = (cdt - vdt) = (c-v)t_0\gamma$. Thus, $\lambda = (c-v)\gamma\lambda_0/c$.

For source/observer moving away from each other, one has the wavelength emitted from the source increasing, thus $\lambda = (cdt + vdt) = (c+v)t_0\gamma$. Thus, $\lambda = (c+v)\gamma\lambda_0/c$.

Where in the last equality in the above, one applies time dilation from special relativity, $t = t_0\gamma$ and the fact that $c = \lambda f = \lambda/t$ in general.

Now that one has the proper battle equipment, one can proceed with the problem.

This problem is essentially the difference in wavelengths seen from a red shift and blue shift, i.e., light moving towards and away from the observer.

$\Delta\lambda = (2v)\gamma\lambda_0/c \Rightarrow v = \frac{\Delta\lambda c}{2\gamma\lambda_0} \approx \frac{\Delta\lambda c}{2\lambda_0} \approx \frac{1.8E-12\times 3E8}{244E-9} \approx 2E3$, where the approximation $\gamma \approx 1$ is made since one assumes the particle is moving at a non-relativistic speed.

2 km is closest to choice (B).

Problem 61 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=61*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Gauss Law*

Recall Gauss Law $\vec{E}\cdot d\vec{A} = q/\epsilon_0$. Thus, $E(4\pi r^2) = \int_0^{R/2} Ar^2 4\pi r^2 dr = 4\pi A(R/2)^5/5$. Solving for E, one has $E = A/5(R/2)^3$, as in choice (B).

Problem 62 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=62*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Capacitors*

Initially, one has the two capacitors connected in parallel, so that each receives the same voltage from the battery. Thus $V = 5 = Q_1/C_1 = Q_2/C_2$. $C_1 = 1$ and $C_2 = 2$, thus $Q_1 = C_1V = 5$ and $Q_2 = C_2V = 10$.

After the battery is removed and the capacitors are re-connected so that the opposite plates face each other, one has (immediately) $Q_1 - Q_2 = -5$. The charges would then redistribute themselves so that the voltage across each capacitor is the same. Thus, denoting the final charge on each capacitor as q_1, q_2, respectively, one has (from charge conservation) $-5 = q_1 + q_2$. Applying the equi-voltage condition, one has $q_1/C_1 = q_2/C_2 \Rightarrow q_1 = q_2C_1/C_2$. Plug that into the charge conservation equation to get $-5 = (1 + C_1/C_2)q_2 \Rightarrow q_2 = 3.33 \Rightarrow V = q_2/C_2 \approx 1.7$, as in choice (C). (As an exercise, one can also check by computing the charge for the other capacitor.)

Problem 63 — *http://grephysics.net/disp.php?yload=prob&serial=2&prob=63*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Particle Physics*

If one knows little about particle physics, one can make (at least) the following deductions, wherein one recalls the composites of each particle:

(A) A muon is a lepton. Leptons, along with quarks, are considered the fundamental particles.

(B) Pi-Meson consists of a quark and its antiparticle. (Contribution to this part of the solution is due to user danty.) Moreover, a pi-meson is a hadron. Hadrons interact with the strong-force, and all of them are composed of combinations of quarks. (The fundamental particles are classified as quarks and leptons.)

(C) A neutron is made up of 3 quarks.

(D) A deuteron consists of a proton and a neutron. (tritium is two neutrons and a proton, while regular Hydrogen is just an electron and proton)

(E) An alpha particle consists of electrons and protons and neutrons.

Choice (A) remains. Choose that.

If one has some time, one might want to remember the elementary particles involved in the Standard Model. There are six quarks and six leptons. Three of the leptons are neutrinos and the other three are the electron, the tau, and the muon. (Also, in a decay similar to beta-decay, a muon is emitted instead of an electron. Charge conservation works since a muon is like an electron except it is about 200 times more massive.)

Wikipedia has a good reference on this:

`http://en.wikipedia.org/wiki/Fundamental_particle`

Problem 78 in GR0177 also has a good extended review of this:

`http://grephysics.net/disp.php?yload=prob&serial=3&prob=78&Submit=Go%21`

Problem 64 ______ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=64*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Nuclear Physics*

In symmetric fission, the change in kinetic energy is just the change in binding energy. The change in binding energy for a $N - nucleon\ heavy$ nucleus is the difference in energy between the initial un-fissioned heavy nucleus and the final 2 medium-sized nuclei,

$\Delta E = 2 \times 0.5N \times 8MeV/nucleon - N \times 7MeV/nucleon = N \times 1MeV/nucleon.$

For a heavy nucleus, one has $N \approx 200$, and thus one arrives at choice (C). (This is due to David Schaich.)

Problem 65 ______ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=65*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Conservation Laws*

From conservation of momentum, one has $mv_0 = Mv \Rightarrow v_0 = Mv/m$. The man does work on both himself and the boat. Thus, the work-kinetic energy theorem has $W = \Delta K = 1/2mv_0^2 + 1/2Mv^2 = 1/2(M^2/m + M)v^2, as in choice (D)$

Problem 66 ______ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=66*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Effective Potential*

One can solve this problem by remembering the effective potential curve $V_{eff}(r) = V(r) + L^2/(2mr^2)$. For the gravitational potential, one has $V(r) \propto -1/r$.

The total energy of the spaceship is $E_s = 1/2m(1.5v_J)^2 + V_{eff}$, while the total energy of Jupiter is $E_J = 1/2m(v_J)^2 + V_{eff}$

(A) A spiral orbit occurs when $E < V_{min}$, which corresponds to $v_s << v_J$.

(B) A circular orbit occurs only when $E = V_{min}$. Since the energy of Jupiter is greater than that of the spaceship–and (see below) since Jupiter itself has $E > V_{min}$, the spaceship must have $E > V_{min}$.

(C) An ellipse occurs for $V_{min} < E < 0$. Planets orbit in ellipses. However, since the speed of the ship is greater than Jupiter's orbit speed by a good bit, one assumes its total energy is $E > 0$.

(D) A parabolic orbit occurs for $E = 0$. The condition is much too stringent.

(E) A hyperbolic orbit occurs for $E > 0$. See (C). Since $E_s > 0 > E_J$, this is it.

Problem 67 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=67*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Schwarzchild Radius*

Using quite a bit of handwaving, the current author has seen an astrophysicist derive the Schwarzchild Radius (radius at which the curvature of space mooches and eats up light completely) via $1/2mc^2 = GMm/r$. (The guy totally neglected relativity, assuming that kinetic energy has the same form in the relativistic regime, but anyway...)

Handwaving like a good astrophysicist, one finds that $r = 2GM/c^2 \approx 2 \times 7E-11 \times 6E24/9E16 = 1cm$.

(Note: the author is currently declared as an astrophysics major, and if the above comment is to be interpreted as pejorative towards astrophysicists, then she has thus implicitly insulted herself.)

Problem 68 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=68*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Lagrangians*

The potential energy of the mass is obviously $U = mgs\cos\theta$, and thus one eliminates choices (B) and (C). (To wit: $L = T - U$, (C) has the wrong sign).

The translational part of the kinetic energy is easily just $1/2m\dot{s}^2$. The rotational part requires the calculation of the moment of inertia for a point particle, which is just $I = mr^2$, where $r = s\sin\theta$, in this case. Thus, the rotational kinetic energy is $1/2m(s\sin\theta)^2\omega^2$. The only choice that has the right rotational kinetic energy term is choice (E).

Problem 69 *http://grephysics.net/disp.php?yload=prob&serial=2&prob=69*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Ampere Law*

Recall Ampere's Law, $\vec{B} \cdot d\vec{l} = \mu_0 I_{in}$.

Since the region one is interested in is a vacuum, one's Ampere Loop encloses all of the current. Thus, the field from each conductor is $B(2\pi r) = \mu_0 J\pi R^2$, where, $I_{in} = J\pi R^2$ and R is the radius of the conductor. (This is a good approximation of the current, as one assumes that the vacuum region in the center is small compared to the area of the conductors.)

Making the approximation that $R \approx d/2$, one has $B = \frac{\mu_0 J\pi R^2}{2\pi R} \approx \frac{\mu_0 J\pi d}{4\pi}$. Since both fields contribute in the center, the field is twice that, $\frac{\mu_0 J\pi d}{2\pi}$, as in choice (A).

(Also, one can immediately eliminate all but choices (A) and (B) by the right-hand rule. One seeks which claims a +y-direction field.)

Problem 70 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=70*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Larmor Formula*

The Larmor formula for power radiated by an accelerated charge is related to the charge and acceleration as $P \propto a^2 q^2$.

The problem gives the following:

A: $q, b, a \Rightarrow P_A \propto q^2 a^2$

B: $2q, 3v, 4a \Rightarrow P_B \propto (2q)^2 (4a)^2 = 64 q^2 a^2$

Thus, $P_B/P_A = 64$, as in choice (D).

Problem 71 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=71*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Particle Trajectory*

One can get a reasonable approximation for the deflection angle as follows.

Assuming that there is no magnetic field, one has from the Lorentz force $F = ma = qE = qV/d$, where one neglects gravitational acceleration. The acceleration is constant, and it is $a = qV/(dm)$.

Recalling the baby physics kinematics equation, $y = 0.5at^2 \Rightarrow dy = atdt$ and the fact that $x = L = vt \Rightarrow dx = vdt$ and $t = L/v$, one can calculate the angle as $\tan\theta \approx dy/dx = \frac{atdt}{vdt} = at/v = \frac{qVL}{v^2 dm}$. Take the arctangent to get choice (A).

Problem 72 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=72*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Lab Methods → *Negative Feedback*

Negative feedback, according to Horowitz's *The Art of Electronics*, has to do with canceling out some of the input in the output. Although that might seem like redundantly adding noise to the system, it actually reduces the amplifier's gain, increases stability (by decreasing nonlinearity and distortion).

From that bit of info, two choices remain. Choice (A) and (B). Choose (A) because negating the feedback should not increase the amplitude.

Problem 73 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=73*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Adiabatic Work*

One should recall the expression for work done by an ideal gas in an adiabatic process. But, if not, one can easily derive it from the condition given in the problem, viz., $PV^\gamma = C \Rightarrow P = C/V^\gamma$.

Recall that the definition of work is $W = \int PdV = \int_{V_1}^{V_2} CdV/V^\gamma = -\frac{1}{\gamma-1} C/V^{\gamma-1}\Big|_{V_1}^{V_2}$, which when one plugs in the endpoint limits, becomes choice (C).

Problem 74 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=74*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Entropy*

Recall the definition of entropy to be $dS = dQ/T$. The heat is defined here as $dQ = mcdT$, and thus $S = \int mcdT/T$.

One is given two bodies of the same mass. One mass is at $T_1 = 500$ and the other is at $T_2 = 100$ before they're placed next to each other. When they're put next to each other, one has the net heat transferred being 0, thus $Q_1 = -Q_2 \Rightarrow T_f = (T_1 + T_2)/2 = 300$.

The entropy is thus $S = \int_{T_1}^{T_f} mcdT/T + \int_{T_2}^{T_f} mcdT/T = mc\left(\ln(3/5) + \ln(3)\right) = 2mc\ln 3 - mc\ln 5 = mc(\ln 9 - \ln 5) = mc\ln(9/5)$, as in choice (B).

Problem 75 ———— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=75*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Fourier's Law*

Recall Fourier's Law $q = -k\nabla T$, where q is the heat flux vector (rate of heat flowing through a unit area) and T is the temperature and k is the thermal conductivity. (One can also derive it from dimensional analysis, knowing that the energy flux has dimensions of $J/(sm^2)$)

Fourier's Law implies the following simplification: $q = -k\frac{\Delta T}{\Delta l}$

The problem wants the ratio of heat flows $q_A/q_B = \frac{k_A l_B}{k_B l_A} = \frac{0.8\times 2}{0.025\times 4} = 32/2 = 16$, as in choice (D). (The problem gives $l_A = 4$, $l_B = 2$, and $k_A = 0.8$, $k_B = 0.025$.)

Problem 76 ———— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=76*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Uncertainty*

One can make a good stab at this problem by applying the uncertainty principle.

I. If the average momentum of the packet is 0, then one violates the uncertainty principle. See IV.

II. Maybe.

III. Maybe.

IV. True, recall the *Gaussian* uncertainty principle $\Delta x\Delta k = \hbar/2$.

Since I is false, choices (A), (C), and (E) are out. Choices (B) and (D) remain. Take the conservative approach and choose (B).

Problem 77 ———— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=77*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Operators*

This problem can be solved (without much knowledge of quantum mechanics) by noting the following general arithmetic trick: $ab = \frac{1}{2}\left((a+b)^2 - a^2 - b^2\right)$.

The problem gives the Hamiltonian $H = -JS_1 \cdot S_2$, which has the same form as the arithmetic trick above. Thus, $H = -J\frac{1}{2}\left((S_1 + S_2)^2 - S_1^2 - S_2^2\right)$.

Recalling some basic linear algebra, one can make use of the eigenvalue equations supplied with the problem defining the eigenvalues of the wanted operators, $S_i^2\psi_i = S_i(S_i+1)\psi_i$.

Thus, $\langle H\rangle = \langle\psi_i|H|\psi_i\rangle = -J\frac{1}{2}\left((S_1+S_2)(S_1+S_2+1) - S_1(S_1+1) - S_2(S_2+1)\right)$, where one has applied the eigenvalue equation above and generalized it for the case $(A+B^2)\psi = (A+B)(A+B+1)\psi$. From a bit of math manipulation, one has arrived at choice (D).

Problem 78 ———— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=78*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Solid State Physics*

A n-type semiconductor is a material with *n*egative-charge carriers, such as electrons. A p-type semiconductor is a material with *p*ositive-charge carriers, such as holes (positrons).

In band theory, n-type semiconductor impurities are (electron) donors, while p-type semiconductor impurities are (electron) acceptors.

The setup is as follows:

Impurities add in more levels to the energy bands. Without impurities, one has just a valance band and a conduction band with an energy gap in between. The impurities supply an extra energy level in between the conduction and valance bands. In an n-type semiconductor, the material becomes conducting when there are electrons in the conduction band; the impurity helps the material become conducting by supplying it with electrons.

Essentially, one starts with a lattice of pure semiconductor atoms, say Silicon. Silicon has four valance electrons and forms a decent crystal lattice. Pluck out a few silicon atoms and replace them with some impurities, like Arsenic, which five valance electrons. The extra electron from each impurity atom is free to roam around. In fact, these extra electrons act as *donors* to the conduction band. This is choice (E).

Problem 79 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=79*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Specific Heat*

The specific heat at constant volume for high temperatures is $c_v = 7/2R$. The specific heat at low temperatures is $3/2R$. Why?

There are three contributions to the specific heat of a diatomic gas. There is the translational, vibrational, and rotational. At low temperatures, only the translational heat capacity contributes $U = 3/2NkT \approx c_v T \Rightarrow c_V = 3/2Nk$. At high temperatures, all three components contribute, and one has $c_V = (3/2 + 1 + 1)Nk = 7/2Nk$.

The general formula is

$c_v = c_v(translational)+c_v(rotational)+c_v(vibrational) = Nk\left(3/2 + 1 + (h\nu/(k\theta))^2 \exp(h\nu/(k\theta))/(\exp(h\nu/(k\theta)) - 1$

Problem 80 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=80*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Wave Phenomena*

There's a long way to solve this problem and then a short. One looks at the choices to find the one that first the physical deduction: when $\mu_l = \mu_r$, the whole incident wave should be transmitted, with 0 reflection. Moreover, in the limit of $\mu_r >> \mu_l$ there should be 0 transmission. Choice (C) is the only one that fits this condition, leading to a ratio of 1 for $\mu_l = \mu_r$.

One can also calculate the exact form of the transmission coefficient for this multi-density string. Take the following,

$$\psi_i = Re\left(e^{-i(k_l x - \omega t)}\right) \tag{37}$$

$$\psi_r = \left(ReRe^{-i(-k_l x - \omega t)}\right) \tag{38}$$

$$\psi_t = \left(ReTe^{-i(k_r x - \omega t)}\right) \tag{39}$$

At the boundary between different density parts, one applies continuity $\psi_i(x=0) + \psi_r(x=0) = \psi_t(x=0)$ to get 1+R=T.

One applies $m\ddot{\psi} = 0 = \frac{\partial\psi}{\partial x}(x < 0) - \frac{\partial\psi}{\partial x}(x > 0)$, where $m = 0$ since there is no point particle situated at the origin, to obtain $ik_l(1 - R) = ik_r T$.

Recalling the nifty relation $\omega = ck$ and $c = \sqrt{F/\mu}$, one solves for T to get $T = \frac{2k_l}{k_r+k+l} = \frac{\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$, as in choice (C).

Problem 81 ________________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=81*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Wave Phenomena → *Beats*

One remembers the relation for beats, i.e., $f_1 - f_2 = f_{beat}$. Beat phenomenon occurs when two waves occur at nearly the same frequency.

Taking $f_0 >\approx 73$ as the fundamental frequency, one deduces the harmonic to be $440/f_0 \approx 6$. $440 - 6f_0 \approx 0.5$, and thus the answer is choice (B).

One can derive the relation for beats by recalling the fact that one gets beat phenomenon when one superposes two sound waves of similar frequency $f_1 \approx f_2$, say, of the form $A\sin(2\pi f_i t)$,

$$f = A\sin(2\pi f_1 t) + A\sin(2\pi f_2 t) = 2A\sin(2\pi(f_1 + f_2)/2t)\cos(2\pi(f_1 - f_2)/2t), \tag{40}$$

where to get beats phenomenon one must have $\cos(t(f_1 - f_2)/2) = \pm 1 \Rightarrow 2\pi = (f_1 - f_2)/2t$, and since there are two beats per period, one has $f_1 - f_2 = f_{beat}$.

Problem 82 ________________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=82*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Optics → *Thin films*

For a thin film of thickness t, one can easily find the condition for interference phenomenon. Since the light has to travel approximately $2t$ to get back to the original incidence interface, one has $2t = m\lambda$. However, since the light changes phase at the interface between air and glass (since glass has a higher index of refraction than air), the condition for constructive interference becomes $2t = m\lambda/2$, where $m \in Odd$.

One can create a table to determine the values of $t = m\lambda/4$.

$$m \Rightarrow t \tag{41}$$
$$1 \Rightarrow \lambda/4 = 122nm \tag{42}$$
$$3 \Rightarrow 3\lambda/4 = 366nm \tag{43}$$
$$5 \Rightarrow 5\lambda/4 = 610nm, \tag{44}$$

and so forth...

One thus finds that choice (E) is correct.

Problem 83 ________________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=83*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Rippled Surface*

The simple intuitive way to solve this is to note that for $d \to \infty$, $v \to 0$, since one gets an infinitely steep (vertical line) hill, and the only way for the particle to stay on the surface (i.e., not accelerate on it) at the vertical drop is if its velocity is 0. The onyl choice with d on the denominator is choice (D).

The more rigorous solution is due to Sara Salha.

Equating centripetal force with gravity at the top of the hill, one has $mv^2/r = mg \Rightarrow v = \sqrt{mgr}$. The non-trivial bit comes from calculating the radius.

Recall the radius of curvature from calculus $1/r = \kappa = \frac{|\ddot{x}\dot{y}-\ddot{y}\dot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}}$. Defining a parameter t as the independent variable, and defining $x = t$, $y = d\cos(kt)$, one finds that $1/4 = \frac{k^2 d\cos(kt)}{1+(kd)^2\sin^2(kt)}$. Evaluate it at $t = 0$ to find $1/r(0) = k^2 d \Rightarrow r = 1/(k^2 d)$, the radius of curvature at the top of the hill. Plug that into the equation for velocity above to get $v = \sqrt{\frac{mg}{kd^2}}$, as in choice (D).

Problem 84 ______ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=84*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Normal Mode*

One can work through the formalism of the usual normal mode analysis or learn how to deal with normal mode frequencies the easy way:

The highest normal mode frequency is due to the two masses oscillating out of phase. The ω^2 contribution from the pendulum is just g/l. The ω^2 contribution from each mass due to the spring is k_i/m_i. This is choice (D).

(If one had to guess, one can immediately eliminate choice (A), since that is the lowest normal mode frequency. In normal modes, there's always an in-phase frequency, which tends to be the lowest frequency.)

Problem 85 ______ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=85*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Wave Phenomena*

One can solve this problem without knowing anything about mechanics (but with just the barest idea of wave phenomenon theory). Following the hint, one considers the limiting cases:

$M \to \infty \Rightarrow \mu/M \to 0$... With an infinite M, the string is basically fixed on the rod, and its wavelength is just $\lambda = L$. One eliminates choice (A) from the fact that $\cos 2\pi = 1 \neq 0$, as μ/M demands in this regime.

$M \to 0 \Rightarrow \mu/M \to \infty$... Without the presence of the mass M, one has $\lambda = 4L$. Thus, $2\pi L/\lambda = \pi/2$. Since $\tan x = \sin x/\cos x$ and $\cos \pi/2 = 0$, one finds that choice (B) is the only one that fits this condition.

Problem 86 ______ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=86*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Particle Trajectory*

There is a force pointing upwards from the Electric field in the y-direction. Suppose the particle is initially moving upwards. Then, the magnetic field would deflect it towards the right... One can apply the Lorentz Force to solve this problem.

If the particle comes in from the left, then the magnetic force would initially deflect it downwards, while the electric force would always force it upwards. Continue applying this analysis to each diagram. It turns out that one has cycloid motion whenever the electric and magnetic fields are perpendicular.

Problem 87 ______ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=87*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Faraday Law*

From Faraday's Law or Len's Law, one has $\vec{E} \cdot \vec{dl} = -d\Phi/dt$. Since, in order for the balls to move, they must move in a circle, one has $dl = 2\pi d/2$; moreover, the induced magnetic field would point in the opposite direction to the field that was before, and one has a current in a loop from the right-hand-rule. The area of the magnetic flux is just πR^2, since the field only goes through the cylindrical region of radius R.

Thus, $E(2\pi d/2) = \dot{B}\pi R^2 \Rightarrow E = \frac{\dot{B}R^2}{d}$.

Now, recall some mechanics. The torque is related to the moment-arm and force by $\tau = \sum \vec{r} \times \vec{F}$, where $\vec{F} = q\vec{E} = q\frac{\dot{B}R^2}{d}$. Since there is a force contribution from each charge, and since, by the right-hand-rule, their cross-products with the moment-arm point in the same direction, one finds the torque to be $\tau = 2(d/2)q\frac{\dot{B}R^2}{d} = dq\frac{\dot{B}R^2}{d} = q\dot{B}R^2$.

Now, recall the relation between angular momentum and torque to be $\sum \tau = \dot{L}$. Replace the $\dot{B} \rightarrow B$ above to get $L = qBR^2$, and so the system starts rotating with angular momentum as in choice (A). (This approach is due to Matt Krems.)

Note that one can immediately eliminate choice (D) since angular momentum is not conserved from the external torque induced (to wit: electromagnetic induction). Moreover, although choice (E) is true in general, it does not apply to this problem.

Problem 88 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=88*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Ampere Law*

Recall Ampere's Law $\vec{B} \cdot \vec{dl} = \mu_0 I_{inclosed}$, where $I_{inclosed}$ is the current enclosed by the loop dl.

Apply it to the region between a and b, $B(2\pi r) = \mu_0 I \frac{\pi r^2 l}{\pi R^2 l} \Rightarrow B = \mu_0 I \frac{r}{2R^2\pi}$, which gives a linearly increasing field, and thus choices (D) and (E) and (A) are eliminated.

Choices (B) and (C) remain.

Apply Ampere's Law to the region outside of the outer sheath. For $r > c$, one has $I_{inclosed} = 0 \Rightarrow B(2\pi r) = 0 \Rightarrow B = 0$. Choice (B) shows the behavior of zero-field outside the sheathed coax cable. Choose that.

Problem 89 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=89*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Electromagnetism → *Trajectory*

The only physics involved in this problem is equating the centripetal force with the Lorentz Force, $mv^2/R = qvB$. The rest is math manipulation and throwing out terms of ignorable order.

The radius of curvature used in the centripetal force equation is given by $R^2 = l^2 + (R-s)^2$, and ETS is nice enough to make this geometry fairly obvious in the diagram enclosed with the original question.

Now, note that since $s << l$, after expanding the expression for r^2, one can drop out terms of higher order. Thus, $R^2 = l^2 + (R-s)^2 = l^2 + R^2 + s^2 - 2Rs \approx l^2 + R^2 - 2Rs + O(s^2)$. Canceling the R's on both side, one finds, $l^2 = 2Rs \Rightarrow R = l^2/(2s)$. Plug this into the force equation above to find,

$$mv/R = qB \Rightarrow 2smv/l^2 = qB \Rightarrow p = mv = qbl^2/2s, \tag{45}$$

which is choice (D).

Problem 90 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=90*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Unscored → *Unscored*

Quote: *This item was not scored.*

Problem 91 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=91*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Thermodynamics → *Second Law*

The Second Law of thermodynamics has to do with entropy; that entropy can never decrease in the universe. One form of it states that *from hot to cold things flow.* A cooler body can thus never heat a hotter body. Since the oven is at a much lower temperature than the wanted sample temperature, the oven can only heat the sample to a maximum of 600K without violating the Second Law. (This solution is due to David Latchman.)

(Also, since the exam is presumably written by theorists, one can narrow down the choices to either (D) or (E), since the typical theorist's stereotype of experimenters usually involves experimenters attempting to violate existing laws of physics—usually due to naivity.)

Problem 92 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=92*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Potential*

Given $V(x) = -ax^2 + bx^4$, one can find the minimum by taking the first derivative (second derivative test indicates concave up), $V'(x) = \left(-2ax + 4bx^3\right)_{x_0} = 0 \Rightarrow x_0 = \sqrt{\frac{a}{2b}}$.

The force is given by $F = -dV/dx = 2ax - 4bx^3$.

The angular frequency of small oscillations about the minimum can be found from,

$$\begin{aligned}
F(x - x_0) &= 2a(x - x_0) - 4b(x - x_0)^3 && (46)\\
&\approx 2ax - 4b(3x_0^2 x) + O(2) && (47)\\
&= (2a - 12bx_0^2)x && (48)\\
m\ddot{x} &= -4ax && (49)\\
\ddot{x} &= -\omega^2 x \Rightarrow \omega^2 = \frac{4a}{m} && (50)
\end{aligned}$$

where one might recall the binomial theorem or pascal's triangle to quickly figure out the trinomial coefficients.

One finds that $\omega = 2\sqrt{\frac{a}{m}}$, as in choice (D).

Problem 93 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=93*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Mechanics → *Potential*

The problem gives a nifty potential energy graph. The period is due to each part of the potential graph.

For the simple harmonic oscillator (SHO) part, one remembers the formula $T = 2\pi\sqrt{m/k}$ (to wit: $m\ddot{x} = -kx \Rightarrow \ddot{x} = -\omega^2 x \Rightarrow \omega = \sqrt{k/m} = 2\pi f = 2\pi/T \Rightarrow T = 2\pi\sqrt{m/k}$). However, that period is for a particle to oscillate from one end of the potential curve to the other end and then back again. Since the graph shows only half of the usual SHO potential, the period contribution from the SHO part should be half the usual period: $T_{SHO_{1/2}} = \pi\sqrt{m/k}$

For the gravitational potential, one can calculate the period from the usual kinematics equation for constant acceleration. Recall the baby-physics equation, $x = \sqrt{1}2gt^2 \Rightarrow t = \sqrt{2x/g}$. The quantity needs to be converted to the relevant parameters of the problem. The problem supplies the constraint that the energy is constant, $E = \frac{1}{2}mv^2 + mgx$. At the endpoint, one has $v = 0 \Rightarrow x = E/mg$. Plugging this into the equation for time, one gets $t = \sqrt{2E/mg^2}$. Since the particle has to travel from the origin to the right endpoint and then back to the origin, the total time contribution from this potential is twice that, $T_{grav} = 2\sqrt{2E/mg^2}$.

The total period is thus the sum of the above contributions, which is choice (D).

Problem 94 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=94*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Statistical Mechanics → *Internal Energy*

The partition function is $Z = \sum e^{-\epsilon_i/kT} = 1 + e^{-\epsilon/kT}$. Internal energy is given by $U = \frac{NkT^2}{Z}\frac{\partial Z}{\partial T} \propto \frac{\epsilon e^{-\epsilon/kT}}{e^{-\epsilon/kT}+1} \propto \frac{\epsilon}{e^{\epsilon/kT}+1}$, as in choice (D).

Problem 95 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=95*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Advanced Topics → *Solid State Physics*

The specific heat of a superconductor jumps at the critical temperature (c.f. with its resistivity jump).

Ordinarily, the specific heat of a metal is $c = aT + bT^3$. When it is superconducting, the first term, the electronic-contribution, is replaced by $\approx e^{-cT}$. The revised plot of the specific heat has a jump from an exponentially increasing specific heat to a much lower value somewhere in the range for positive T.

Reference: Ibach and Luth p 270ff

Problem 96 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=96*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Special Relativity → *Maximal Velocity*

Conservation of momentum yields $p = 2\gamma m_e v$, where p is the momentum of the photon.

Conservation of energy yields $pc = 2\gamma m_e c^2$. Plug in the above equation for momentum to get $2\gamma m_e vc = 2\gamma m_e c^2$. This occurs when $v = c$. Since v is the velocity of the electron, and since according to relativity, only a photon (to wit: a massless particle) can move at the speed of light—one of the conservation laws is not conserved!

This is choice (A).

Problem 97 ______________ *http://grephysics.net/disp.php?yload=prob&serial=2&prob=97*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Probability Density Current*

If one has forgotten the expression for the probability density current, then...

The probability current density can be effortlessly derived from recalling the definition of probability and Schrodinger's Equation—both of which *every* physics (or engineering) major should know by heart.

Probability is defined (in the Born Interpretation) as $P = \int |\Psi(x,t)|^2 dx$. One should recall that $|A|^2 = A^*A = AA^*$ in general (to wit: the absolute value squared of a complex expression is itself times its complex conjugate).

The time-dependent Schrodinger's Equation is

$\hbar i \frac{\partial \Psi}{\partial t} = H\Psi = -\frac{\hbar^2}{2m}\Psi'' + V\Psi$,

where $H = -\hbar^2/2m\frac{\partial}{\partial x} + V$ has the form of the familiar time-independent Hamiltonian. From this, one finds that $\frac{\partial \psi}{\partial t} = \frac{-i}{\hbar}\left(-\frac{\hbar^2}{2m}\Psi'' + V\Psi\right)$.

Generalizing the idea of a current from classical physics to the idea of a probability current, one takes the time derivative of the probability to get $\frac{d}{dt}P = \frac{d}{dt}\int |\psi|^2 dx = \int \frac{d}{dt}|\psi|^2 dx = \int \left(\psi\frac{\partial \psi^*}{\partial t} + \psi^*\frac{\partial \psi}{\partial t}\right) dx$, where the product-rule for baby-math derivatives has been used and the derivative has been taken inside the integral because the integral and derivative are with respect to different variables.

Plugging in the expression for $\dot{\psi}$ from the Schrodinger's Equation, one gets

$dP/dt = \int \left(\Psi\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\Psi^{*''} + V\Psi^*\right) + \Psi^*\frac{-i}{\hbar}\left(-\frac{\hbar^2}{2m}\Psi'' + V\Psi\right)\right) dx$,

where the terms involving V's cancel out, and thus,

$dP/dt = \int \frac{i}{\hbar}\frac{\hbar^2}{2m}\left(-\Psi\Psi^{*''} + \Psi^*\Psi''\right) dx = \frac{i\hbar}{2m}\int\left(-\Psi\Psi^{*''} + \Psi^*\Psi''\right) dx$

Rewriting $\int\left(-\Psi\Psi^{*''} + \Psi^*\Psi''\right) dx = \int \frac{\partial}{\partial x}\left(-\Psi\Psi^{*'} + \Psi^*\Psi'\right) dx$, one can eliminate the integral in the probability current by applying the fundamental theorem of calculus (to wit: $\int_a^b \frac{\partial \psi}{\partial x} dx = \psi(b) - \psi(a)$),

$dP/dt = \frac{i\hbar}{2m}\left(-\Psi\Psi^{*'} + \Psi^*\Psi'\right)$. But, since the probability current is usually define as $dP/dt = J(a) - J(b)$, one has

$dP/dt = \frac{i\hbar}{2m}\left(\Psi\Psi^{*'} - \Psi^*\Psi'\right)$.

(Aside:) One can print-out a cool poster or decent T-shirt iron-on to remember the Schrodinger's Equation (among other miscellanai) at a site the current author made several years ago,

http://anequationisforever.com/ds.php

One can remember the general form of the probability current by recalling that it has to do with the difference of Ψ times its conjugate.

Right, so onwards with the problem:

The problem gives the wave function, so one needs just chunk out the math to arrive at the final answer,

$\Psi' = e^{i\omega t}k\left(-\alpha\sin(kx) + \beta\cos(kx)\right)$

$\Psi^{*'} = e^{-i\omega t}k\left(-\alpha\sin(kx) + \beta\cos(kx)\right)$

Thus,

$\Psi^*\Psi' = k\left(\alpha^*\cos(kx) + \beta^*\sin(kx)\right)\left(-\alpha\sin(kx) + \beta\cos(kx)\right)$ and,

$\Psi\Psi^{*'} = k\left(\alpha\cos(kx) + \beta\sin(kx)\right)\left(-\alpha^*\sin(kx) + \beta^*\cos(kx)\right)$, where one notes that the imaginary terms go to unity from the complex conjugate.

Plugging this into the probability current, one arrives at the expression for choice (E).

Problem 98 ——— *http://grephysics.net/disp.php?yload=prob&serial=2&prob=98*

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Symmetry*

One recalls the simple harmonic oscillator wave functions to be symmetric about the vertical-axis (even) for the 0th energy level, symmetric about the origin (odd) for the first energy level, and so on.

If there is a wall in the middle of the well, then all the 0th energy level wave function would disappear, as would all even wave functions.

Recall the formula for SHO $E = \hbar\omega\left(n + \frac{1}{2}\right)$. The first few odd states (the ones that remain) are $E_1 = 3/2\hbar\omega, E_3 = 7/2\hbar\omega$, etc. This is choice (D).

Problem 99 $http://grephysics.net/disp.php?yload=prob\&serial=2\&prob=99$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Two-State Systems*

Recall the mnemonic for remembering what a LASER is—Light Amplified Stimulated Emission Radiation.

A laser consists of two states, with a metastable-state in between the top and bottom state. Initially, one has all the atoms in the ground-state. But, photons come in to excite the atoms (through absorption), and the atoms jump into the top state; this is called a population inversion, as the ground-state atoms are now mostly in the top "inverted" state. More photons come in to excite these already excited atoms, but instead of absorption, emission occurs, and the atoms jump to a lower meta-stable state while emitting photons (in addition to the incident photons). The atoms stay in this metastable state due to selection rules, where a transition back to the ground-state is forbidden.

One doesn't need to know all that to solve this problem. Instead, merely the idea of a laser requiring two main states and a metastable state in between would suffice. Since the question gives the bottom state as $n = 1$ and top state as $n = 3$, one deduces that the metastable state must be $n = 2$, as in choice (B).

Problem 100 $http://grephysics.net/disp.php?yload=prob\&serial=2\&prob=100$

For typo-alerts, alternate solutions, and questions pertaining to this problem—check out this link! ↑

Subject Type

Quantum Mechanics → *Raising Operator*

$a^\dagger = a^* = \sqrt{\frac{m\omega_0}{2\hbar}}\,(x + -ip/(m\omega_0)) \neq a$, and thus choice III must be true. This eliminates all but choice (C) and (E). Since one knows that the raising operator acts to raise the energy level, $[a, H] \neq 0$ implies that they don't commute. This leaves just choice (C).

(Choice II is false because, from above, the condition for a Hermitian operator is violated $a^\dagger \neq a$.)