11 Crushing and classification

Estimates vary, but it is generally accepted that of all the energy used in the World something like between 1 and 10% is in *comminution*, i.e. the processes of crushing, grinding, milling, micronising, etc. Changing the size of the particles by crushing creates many important industrial products. An example is sugar, which has three different grades: granular, castor and icing; chemically they are the same material the main difference is their particle size. So, grinding followed by *classification* is all that is needed to produce the three different products.

Historically, grinding became a mechanised industrial process with the advent of water and wind powered mills to process wheat, barley. animal feed, etc. The mills used a flat stationary stone with a moving mill wheel revolving on-top. A derivative of this type of mill, an edge-runner mill, can still be found in use today, albeit electrically driven. In old mills classification was important to separate the flour from the husk, this was often achieved by sieving. Modern mills often combine classification and milling within the same device by having an up-draught to carry the finer particles away from the milling section. These are known as air swept devices.

Milling of minerals has been an important part of the recovery of metals and industrial minerals for many centuries. Often a mineral of interest is surrounded by rock of a different type, which may be worthless; i.e. a *gangue* mineral. The grain boundary between the desired mineral and the gangue will be the weakest mechanical point and the most likely to break. Thus, grinding to the *liberation size* will release the valuable mineral so that it may then be separated from the gangue. This is a process that is employed for metal ore mining as well as precious minerals recovery – see the box below Table 11.1. In the table the Moh's scale of hardness is shown. It is based on the mineral lower in the table being able to scratch the mineral above. 'Soft' minerals are 1 to 3, 'medium' are 4 to 6 and 'hard' are 7 to 10. However, the hardness value is entirely arbitrary, it is merely a ranking of the described minerals. The table also includes the Bond Work Index, which is discussed in the next section.

An initial size reduction from material several centimetres in diameter down to one centimetre, or so, is often termed *primary crushing*. This may be followed by further size reduction, *secondary crushing*, and then *pulverising*, or *fine grinding*. The term *micronising* has become popular for the reduction of particle size to this dimension. Thus, coal being fed into a pulverised fuel burner for electricity generation will undergo the first three grinding operations. Primary crushing normally takes place close to the mine head and relies on equipment with very large throughputs, usually having two surfaces approaching and retreating from each other. Examples

Classification is the term for industrially sorting particles into different size fractions or *grades*.

Table 11.1 Moh’s scale

Material	Moh's hardness	Bond Work index (kWh ton^{-1})
Talc	1	
Gypsum	2	7
Calcite	3	
Fluorite	4	10
Glass	Scratches above	11
Apatite	5	
Orthoclase Feldspar	6	12
Quartz	7	14
Topaz and beryl	8	
Carborundum	9	26
Diamond	10	

Diamonds are processed by crushing the rock to *liberate* the diamond (hopefully the diamond doesn't crush sometimes it does). One big diamond is worth a lot more than two small ones!

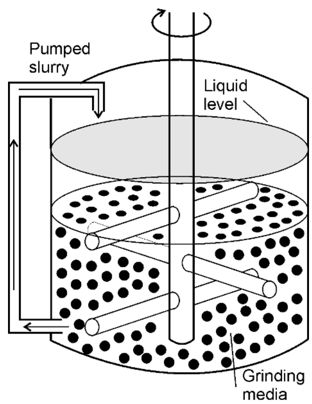


Fig. 11.1 An attritor mill

include *Jaw* and *Cone Crushers*. Secondary crushing can be by rotating surfaces such as swing hammer mills, for brittle materials, and roll crushers. Finer grinding usually takes place in rotating vessels, such as ball and rod mills. Very fine grinding, to sizes less than 10 μm, requires high energy and attrition mills are often used. These have much lower throughputs than the primary crushers and an example of an attrition mill is illustrated in Figure 11.1. These are usually found in the fine chemical industry. Examples of these and other types of machines can be easily found on The Internet.

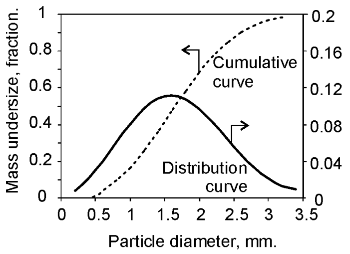
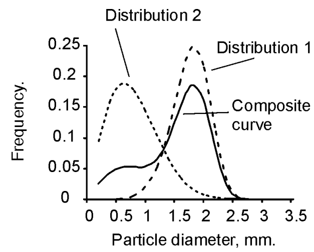


Fig. 11.2 Plastic particles before grinding

Experimental observations suggest that particles may undergo elastic cracking and viscoelastic cracking. In the former case the crack propagates around curves like onion peels and high longitudinal tensile stress is displayed. Viscoelastic is a term used to imply properties of both a liquid and a solid and such cracking propagates through a particle like orange slices and the longitudinal strain is compressive.

The different type of fracture mechanisms may be observed from the particle size distribution functions of a mill product. Figure 11.2 provides the particle size distribution of a mixture of two plastic waste products. After grinding, the particle size distribution changed to that shown in Figure 11.3. Clearly, the two different types of plastic displayed different behaviour during crushing. Both were crushed: the coarse end of the distribution shown in Figure 11.2 has been removed, but the effectiveness of the crushing depended on the material processed. Hence, two separate distribution curves can be deduced from the composite mill product distribution curve. The two separate distribution curves were deduced from the Rosin-Rammler Bennett (RRB) cumulative function, equation (2.5)



	$x_{63.2}$	n
RRB Distribution 1	1.9	6.9
RRB Distribution 2	0.9	2
Fraction of 2:	0.27	

Fig. 11.3 Plastic particles after grinding – two separate distributions added together to provide the composite curve with RRB model parameters

$$N_3(x) = 1 - \exp\left[-\left(\frac{x}{x_{63.2}}\right)^n\right]$$

where $x_{63.2}$ represents the particle size at which 63.2% of the distribution lies below and n is a constant called the uniformity index. Hence, two separate curves were deduced for the crushed plastics and they were added together in the mass ratio of 0.27:0.73 from distributions 2 and 1, respectively. The different crushing behaviour of mixed plastics is one way in which they may be separated for recycling. Also, their behaviour could be changed by the temperature employed during crushing.

11.1 Energy utilisation

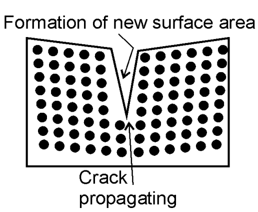


Fig. 11.4 Crack propagation in brittle fracture

If a particle undergoes brittle fracture a crack should propagate through the particle creating new surface area, see Figure 11.4. There will be a corresponding increase in the surface energy (γ – measured in the units of J m^{-2}). The relation between the tensile stress (σ_B), crack length (c_L), Young's modulus (E) and surface energy is

$$\sigma_B = (2\gamma E / \pi c_L)^{1/2} \tag{11.1}$$

Efforts to relate the increase in the surface area to the energy required for the crushing have met with very limited success. The theoretical energy required in comminution to create a new surface is often less than 1% of the required amount – a lot of energy goes into effects other than the new surface itself. During crushing, energy is used by the processes of: *elastic* deformation (not including breakage) of the particles, *inelastic* deformation, *elastic* deformation of equipment, friction between particles, friction between particles and machinery, *noise*, heat and vibration, and friction losses in equipment drives. Together with the energy required to form a new surface.

Noise can be used to control the mill - an empty mill 'rings'.

Elastic deformation returns to the previous shape but an *inelastic* particles bends out of shape or flattens like putty.

11.2 Crushing laws

All the common basic equations relating energy requirement for comminution can be derived from a single ordinary differential equation

$$\frac{\mathrm{d}E}{\mathrm{d}L} = -kL^m \tag{11.2}$$

where E is power, L is particle dimension (diameter), k is a constant and m takes one of three values depending on the particle size: -2, -1.5 or -1. The resulting equations are due to: Rittinger, Bond and Kick, respectively.

Exercise 11.1
Using equation (11.2) derive the three crushing law equations by substituting in n and integrating.

Rittinger (1867) postulated that the energy per unit mass required is proportional to the new surface area produced, i.e. energy = (final surface area - initial surface area). Using an energy per unit mass basis, where

surface area $= f_{sa}\, x^2$
mass $= f_v\, x^3$
Then,

$$E \propto \left[\frac{x_f^{\,2}}{x_f^{\,3}} - \frac{x_i^{\,2}}{x_i^{\,3}}\right] \text{ or } E = k_R\,(S_{\text{vfinal}} - S_{\text{vinitial}}) \tag{11.3}$$

where x_f and x_i are the final and initial diameters. Kick (1885) assumed that the energy required is equal to the reduction ratio based on the ratio of the volume before to the volume after crushing. Thus, the energy required is directly proportional to the size reduction, e.g. the same amount of energy is required to go from 2 to 1 cm as from 1 to 0.5 cm.

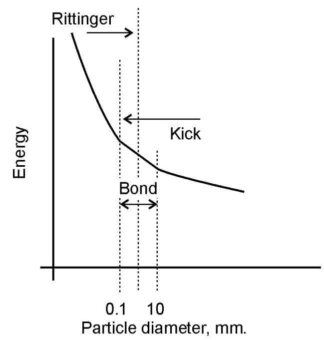


Fig. 11.5 Crushing laws

$$E = k_K \ln\left(\frac{x_f}{x_i}\right) \tag{11.4}$$

Bond (1952) suggested that the total work required to break a cube of size x is proportional to x^3. On fracture and creation of the first crack the energy becomes proportional to x^2, so it was argued that for irregular particles the energy must lie between x^2 and x^3 and a value of $x^{5/2}$ is taken. Again on an energy per unit mass basis and the x_f=$x_{80\%}$ value

The $x_{80\%}$ value is the particle diameter where 80% of the distribution lies below on the cumulative mass undersize curve.

$$E \propto \left[\frac{x_f^{5/2}}{x_f^{3}} - \frac{x_i^{5/2}}{x_i^{3}} \right]$$

$$\text{giving } E = W_i \left[\frac{10}{\sqrt{x_f}} - \frac{10}{\sqrt{x_i}} \right] \qquad (11.5)$$

where W_i is the Bond Work Index, again in energy per unit mass terms. The index is defined as the energy required to crush from infinite size down to 100 μm, hence the 10 inside the brackets. Bond's Work Index values, in kWh per *short* ton: i.e. 2000 lbs or 907 kg, roughly follow the Moh's scale of hardness, see Table 11.1.

In Summary, Kick's law is better for larger particles and Rittinger's for fine grinding, see Figure11.5. Large particles are easier to break than smaller ones as they have more cracks and faults. Often the *laws* are used in conjunction with empirically given constants: it can be dangerous to use the *book* values without some test work.

11.3 Breakage and selection functions

The *Breakage function*, B(y,x), is the fraction by mass of breakage products from size x that fall below size y, where $x \geq y$. The *Selection function*, S(x), is the fraction by mass of particles that are selected and broken in time t – sometimes referred to as the specific rate of breakage and related to revolutions of the mill.

Remember that $N_3(x)$ is the cumulative mass fraction below size x, and we can define the mass of the mill charge as W. Then the amount broken below y in time dt is:

comminution mechanisms
compression: between two solid surfaces,
attrition & impact: against a solid surface and other particles,
cutting: of the particles,
shear: against surrounding fluid, particles and surfaces,
non-mechanical: e.g. laser and plasma ablation.

$$W\mathrm{B}(y,x)\frac{\mathrm{d}N_3(x)}{\mathrm{d}x}\mathrm{d}x\mathrm{d}t \qquad \text{where} \quad \frac{\mathrm{d}N_3(x)}{\mathrm{d}x} = n_3(x)$$

in general, $N_3(x)$ and $n_3(x)$ are functions of x and t, call these $N_3(x,t)$ and $n_3(x,t)$. Concentrating on size y in the mill, noting the definition of breakage function, then at t=0 W=W_0 (the original mill charge) and using F for mass feed rate and R for mass removal rate then:
mass in mill below size y at time t is =
original mass + mass added - mass removed + mass created from sizes above y and =f[S(x) & B(y,x)]
Algebraically this is represented as:

$$WN_3(y,t) = W_0(N_3(y,0)) + \int_0^t [N_3(y,t)]_F\, F dt - \int_0^t [N_3(y,t)]_R\, R dt ...$$

$$+ \iint \frac{\partial N_3(x,t)}{\partial t} \mathrm{S}(x)\mathrm{B}(y,x)\mathrm{d}x\mathrm{d}t \qquad (11.6)$$

In practice, it is difficult to distinguish between the breakage and selection functions, both are measured together. Tests have been tried with single particles and closely sized mill feeds. For most calculations it is necessary to assume S(x)=1.

There are many factors that influence the breakage and selection functions for particles, these include: toughness, abrasiveness, feed size, cohesiveness and adhesion, particle form and structure, softening and melting, organic content, particle size distribution, bulk density and agglomeration. In addition the following factors need to be considered to ensure safe operation: toxicity, explosion and fire hazard. Some of these properties may be favourably changed by milling cryogenically, to encourage brittle fracture, and this is common for plastics. Milling under an inert atmosphere is common for explosive materials and ones that may spoil by exposure of fresh surfaces to air.

11.4 Milling circuit matrix

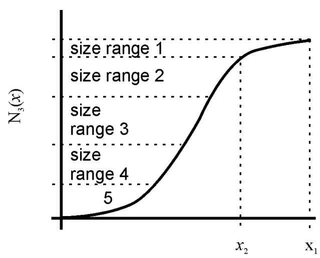
Broadbent and Callcott introduced a breakage function of the form

$$B(y,x) = \frac{\exp(y/x) - 1}{\exp(1) - 1} \qquad (11.7)$$

For example, taking the milling of material less than 1000 μm it is possible to define five grades: 1000 to 800, 800 to 600, 600 to 400, 400 to 200 and material all less than 200 μm. Hence, the grade boundaries (y) will be: 800, 600, 400 and 200 μm. We can define the fraction by mass in these grades before milling as: x_1, x_2, x_3, x_4 and x_5. The breakage function for grade 1 (1000 to 800 μm) with y=800 μm will provide the fraction, by mass, of material starting in this grade and ending up below 800 μm. In order to apply equation (11.7) a single value must be selected to represent the grade, which is usually the mid-point; i.e. 900 μm for grade 1 above. The fraction by mass of material after milling in grade 2 (800 to 600 μm), and originating from grade 1, will be the amount calculated above *minus* the amount passing through the grade 2 to 3 boundary; i.e. the breakage function with x=900 μm and y=600 μm. All the resulting fractions by mass of material starting in grade 1, and now residing in grades 1 to 5, must be equal to unity; to ensure conservation of mass. This logic is illustrated in Figure 11.6.

The example provided above only considers the milling of the size grade 1, i.e. 1000 to 800 μm. Size grade 2 will also be milled, resulting in breakage products in the grades 600 to 0 μm together with unbroken material remaining in grade 2: 800 to 600 μm. The same logic can be applied, using equation (11.7), but with a value of x=700 μm to represent this grade. Likewise grades 3 and 4 must be considered this way. However, *all* the material entering the mill that is less than 200 μm will leave the mill still less than 200 μm; hence, the breakage function will be unity for grade 5. It should be evident that, as this is a milling operation and not an agglomeration, the fraction by mass starting in a lower sized grade and reporting, after milling, to a higher sized grade will be zero. If we represent the mill product by p and the mill feed by f, in mass flow rates, then the material breaking from the top size and remaining in the top size grade will be

$$p_1 = b_{11} f_1$$



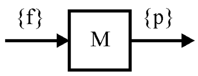
$$b_{11} = 1 - B(x_2, x_1)$$
$$b_{21} = B(x_2, x_1) - B(x_3, x_1)$$
$$b_{31} = B(x_3, x_1) - B(x_4, x_1)$$
$$b_{41} = B(x_4, x_1) - B(x_5, x_1)$$
$$b_{51} = B(x_5, x_1)$$

$$\sum_1^n = 1$$

i.e. conservation of mass

Fig. 11.6 The breakage function and conversion into amounts entering differing grades after milling where b_{11} was defined in Figure 11.6. Likewise, the mass flows entering all the other grades will be

$$p_2 = b_{21} f_1\,; \qquad p_3 = b_{31} f_1\,; \qquad p_4 = b_{41} f_1\,; \qquad p_5 = b_{51} f_1$$



i.e.:

$p_1 = b_{11} f_1$
$p_2 = b_{21} f_1 + b_{22} f_2$
$p_3 = b_{31} f_1 + b_{32} f_2 + b_{33} f_3$
$p_4 = b_{41} f_1 + b_{42} f_2 + b_{43} f_3 + b_{44} f_4$
$p_5 = b_{51} f_1 + b_{52} f_2 + b_{53} f_3 + b_{54} f_4 + b_{55} f_5$

italicised values represents amount remaining unbroken within grade

Fig. 11.7 Mill product and feed material balance

The full mass flow rates for the mill product, and a schematic representation of it, are illustrated in Figure 11.7. It is possible to represent this material balance in matrix form, as shown below

$$\begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & 0 \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{bmatrix} \tag{11.8}$$

The matrices may be represented as follows

$$\{p\} = \mathbf{M}\{f\} \tag{11.9}$$

where **M** is the *milling matrix*. Figure 11.7 illustrates a single pass of material through the mill and it is highly unusual that such a single pass would produce the desired mill product quality. It is common to recycle mill product back to the mill for a second, or more, chance of breakage. Thus, Figure 11.7 represents what is called *open circuit grinding*. In order to return oversize material back to the mill, but remove undersize material that is our product, a classifier is required. A classifier is schematically represented in Figure 11.8 and the classifier matrix is shown below

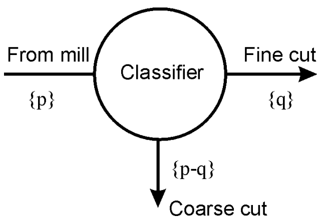


Fig. 11.8 Schematic representation of a classifier

$$C = \begin{bmatrix} c_{11} & 0 & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{33} & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & c_{55} \end{bmatrix} \tag{11.10}$$

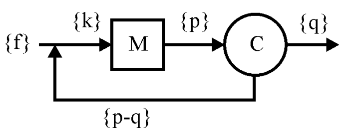


Fig. 11.9 Closed circuit grinding

It is usual to define *grade efficiency for a classifier* as being the fraction, by mass, of material entering the fine cut of the classifier. During classification no alteration of particle size takes place, it simply sorts the particles within a grade: some entering the coarse cut and some the fine. Hence, the classifier matrix is a leading diagonal matrix of fractional values, where the top left element should be a low value and the bottom right element should be approaching unity.

The mill and classifier, with recycle, constitute *closed circuit* grinding and are illustrated in Figure 11.9. At steady state, the mass fed to the circuit {f} must be in balance with the circuit product {q}. The mass flow rate of material being returned to the mill by the classifier is large and the *circulating load* is defined as the ratio of the amount returned {p-q} to the amount fed {p} or {q}. Typical values of circulating loads are in the 100's of percent. Clearly, the circuit product is the classifier matrix operating on the mill product

$$\{q\} = \mathbf{C}\{p\} \tag{11.11}$$

Using the labelling shown in Figure 11.9, the mill product is

$$\{p\}=\mathbf{M}\{k\} \tag{11.12}$$

and the recycle is

$$\{p-q\}=(\mathbf{I}-\mathbf{C})\{p\} \tag{11.13}$$

where **I** is the identity matrix. Considering the mass flow entering the mill, which is the summation of the feed to the circuit and the recycle

$$\{k\}=\{f\} + (\mathbf{I}-\mathbf{C})\{p\}=\{f\} + (\mathbf{I}-\mathbf{C})\mathbf{M}\{k\}$$

and rearranging for {k} using matrix algebra

$$\{k\}=[\mathbf{I}-(\mathbf{I}-\mathbf{C})\mathbf{M}]^{-1}\{f\} \tag{11.14}$$

It is possible to substitute equation (11.14) into equation (11.12), to provide the mill product and then into equation (11.11) to provide the circuit product.

The practical possibilities represented by equation (11.14) are significant. Provided the operating matrices remain constant, i.e. **C** and **M** do not change under different loadings, then using equations (11.11) to (11.14) provides a means to determine what the effect of a change in the feed condition will be on all the mass flow rates throughout the rest of the circuit, including the circuit product. In order to use equation (11.14) the feed column vector {f} must be known, but this is likely to be the case. It represents the feed mass flow rates in the size grades selected: 1000 to 800 μm, etc. in the example quoted. Hence, simulation of the milling circuit under different loading conditions can be provided: covering both a simple increase in the overall feed flow rate into the circuit and the influence of a change in size distribution of the feed. Also, it is possible to check the change in energy requirement by the mill using the matrix modelling: e.g. if Rittinger's constant in equation (11.3) is known, then the specific surface of the feed and mill product may be calculated from the mass flow rates in these streams by dividing the mass flow in each grade by the total mass flow, to give the mass fraction, and then using equation (2.17) for specific surface. More complicated mill circuits can be modelled in a similar way, the objective is always to relate the column vectors within the circuit to the feed column vector, using only the operating matrices that are assumed to remain unchanged under the different operating conditions.

11.5 Population balances

The milling matrix approach described above is a convenient method of conducting a material balance within a particle technology unit operation. It is possible to write a set of simultaneous algebraic equations to perform the same task, but the matrix format is simpler. Also, computer spreadsheets provide all the required matrix functionality required to solve the equations, including matrix inversion. The generic term for the material balancing described above is a *population balance*. Splitting the distribution into separate grades enables the investigation of each grade separately from its

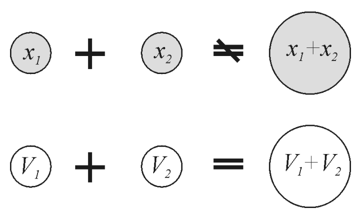


Fig. 11.10 Two particles merged: volumes are additive but not diameters

neighbours to be performed. Thus reactivity, or physical processing characteristics, that are specific to that grade can be applied. Further examples of unit operations that can apply this method include: crystallisation, agglomeration, particle production by atomisation and attrition in fluidised beds. In many cases both particle break-up and formation have to be considered simultaneously.

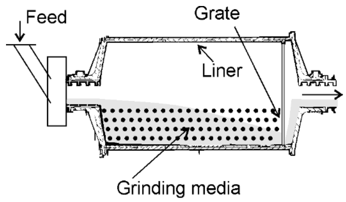


Fig. 11.11 Illustration of a ball mill with an attached grate discharge to retain balls and oversize material

Care has to be exercised over some population balance processes, as illustrated in Figure 11.10. If two particles are completely merged together it is reasonable to assume that the resulting volume will be the same as the volume of the two particles added together. However, the resulting particle diameter is not the same as the two diameters added. Thus, it has been argued that a more complete description of the breakage function should be based on volumes, and not particle diameters. The *deaths* of particles within a given volume increment D(V) is represented by

$$D(V) = S(V)n_o(V) \tag{11.15}$$

where S(V) is the selection function and $n_o(V)$ is the number of particles per unit volume. All the terms in equation (11.15) are volume increment dependent; so, (V) is used. The *births* B(V) are provided by particles from sizes above breaking and creating products that lie within the volume range currently considered

$$B(V) = \int b(V_1, V_2)D(V_1)dV \tag{11.16}$$

where the breakage function has been modified for particle volume rather than diameter. Equation (11.15) can be introduced into (11.16) to provide an expression for the births in terms of both the particle breakage and selection functions. Numerical values may be assigned to the selection function based on the assumption that it is proportional to the particle volume.

The birth and death rates may be combined with the appropriate flow particle equations for the system under consideration; e.g. once through or recycle, in order to conduct a complete material balance based on the usual input-output=accumulation.

11.6 Summary

The production of particles by crushing and grinding is a very important operation throughout the world. In most cases mechanical classification is also required, to return the oversize material to the mill. Common classification equipment includes sieves, or when used on an industrial scale these are called *screens*. In some cases decks of screens may be used to fractionate the product: the largest screen size on the top deck. Slotted screens are often used to minimise screen *blinding*, or blockage. Alternative classifier designs include cyclones and hydrocyclones for dry and wet milling operations respectively, and wet classifiers using screws and rakes to lift the coarser particles to one end, whereas the finer particles leave with the liquid flow at the other. In some cases the screen can be bolted onto the mill itself;

Grinding media

Common grinding media such as balls in a ball mill, or attrition mill, include:
alumina (ceramic),
carborundum,
silica,
zirconia,
nylon, and
steel.

for example, on a ball mill the feed enters at one end and the product may leave over an attached screen, called a *trommel*, at the other. Grates, or screens, may be used within the mill to retain the grinding media and some of the oversize, see Figure 11.11.

No mechanical classification will be perfect, normally the *separation curve*, see Chapter 14, is very shallow. Hence, a significant amount of fine material is usually recycled to the mill and oversize material may be found in the circuit product. For critical applications it may be necessary to classify the product for a second, or more, times to remove oversize. One of the most fundamental decisions to be made is whether to wet or dry mill a material. In most cases a dry product is required, but dry milling very fine material (<20 μm) is not efficient. The dry powder tends to coat the grinding media and vessel. Wet milling provides a viscous medium that absorbs some of the grinding energy, but it does help to disperse the fine particles so that they can receive impact, shear and abrasion. However, the need for wet milling, or otherwise, is a material dependent property and should be considered for each case individually.

11.7 Problems

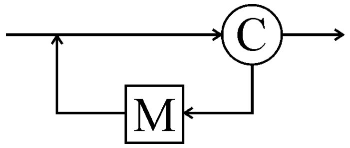
1. Rock is crushed in a cone crusher. The feed is a nearly uniform 50 mm sphere. The screen analysis of the product is given below. The power required to crush this material is 0.429 MW/tonne, of this 11.18 kW is required to drive the empty mill. By reducing the clearance between the crushing head and the cone, the screen analysis of the product becomes finer, i.e. second grind product.

Tyler Mesh size	Mesh size in μm	First grind product (%)	Second grind product (%)
-4 +6	4699 to 3327	3.1	-
-6 +8	3327 to	10.3	3.3
-8 +10	2362 to	20.0	8.2
-10 +14	1651 to	18.6	11.2
-14 +20	1168 to	15.2	12.3
-20 +28	833 to	12.0	13.0
-28 +35	589 to	9.5	19.5
-35 +48	417 to	6.5	13.5
-48 +65	295 to	4.3	8.5
-65		0.5	-
-65 +100	208 to	-	6.2
-100 +150	147 to	-	4.2
-150	104	-	0.3

a) Using Rittingers Law, what do you estimate the power requirement for the second grind to be?

Rittingers Law
hint: use energy per unit mass is proportional to change in specific surface

b) Using Bond's method, estimate the power necessary for each grind, refer to Table 11.1 for a suitable Work Index.



2. A mineral has the following breakage function

$$B(y,x) = \frac{\exp(y/x) - 1}{\exp(1) - 1}$$

and is being classified and crushed in the circuit shown left. Where **M** is the mill and **C** is the classifier. The feed to the circuit is:

cumulative mass undersize (%):	100	85	24	10
particle diameter (μm):	1000	750	500	250

and the feed rate into the circuit is 5 tonnes per hour.

i) Construct a mill matrix based on the above diameters using the given breakage function.
ii) If the classifier can be represented by a leading diagonal matrix of elements: 0.1, 0.3, 0.5 and 0.7, what will be the size distribution and total flow rate of the coarse cut from the classifier?

3. i) When operating a mill in closed circuit show how it is possible to represent the stream entering the mill (k) as
$k = \boldsymbol{a}f$
where f is the column vector representing the feed entering the circuit and $\boldsymbol{a}$ is an operating matrix.
ii) A mill requires 0.4 MW/tonne when the operating matrix $\boldsymbol{a}$, milling matrix and circuit feed column vector are

$$\begin{pmatrix} 1.923 & 0 & 0 \\ 0.361 & 1.563 & 0 \\ 0.025 & 0.017 & 1.111 \end{pmatrix}; \begin{pmatrix} 0.6 & 0 & 0 \\ 0.3 & 0.9 & 0 \\ 0.1 & 0.1 & 1 \end{pmatrix}; \begin{pmatrix} 2 \\ 1 \\ 0.1 \end{pmatrix}$$

where the size ranges corresponding to the rows are 5000 to 2000, 2000 to 1000 and 1000 to 0 μm, and the feed column vector rows are in tonnes per hour. Calculate the empirical constant in Rittinger's equation in the units of metres Mega-Watts per tonne, and the circulating load. If the circuit feed rates in each grade were to change explain briefly how the milling model and energy equation may be used to predict circuit performance and mill energy requirement. Suggest other important factors that would influence the actual mill power requirement if the feed rate was changed.

4. Granulated sugar is sieved at 430 μm in sieve A. The oversize is milled and the product sieved in an identical sieve B. The oversize from this sieve is recycled to the mill. The undersize is combined with the undersize from sieve A and is further sieved at 250 μm to yield two products: a fine cut which is icing sugar and a coarse cut which is castor sugar. Sketch the circuit and obtain an expression for each product assuming that the mill is represented by matrix **M**, sieves A and B by matrix **C** and the final sieve by matrix **D**.

Note, this question expects a series of operator matrices and the only column vector should be for the feed.