Using TI-Nspire to Scaffold the Development of Teacher Questioning Skills That Promote Students' Understanding of Introductory Concepts of Rates of Change

Preliminary Draft Report for Texas Instrument's TI-Npsire Seed Funding

Curriculum Research & Development Group
University of Hawaii

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The Curriculum Research & Development Group, College of Education, University of Hawaii conducted a Phase I study that used TI-Nspire with lessons from the Course 4 (Grade 12), Unit 1 of the Core-Plus Project, Contemporary Mathematics in Context. This introductory unit develops students' understanding of the fundamental concepts underlying calculus and their applications. This NSF-funded reform curriculum is noted for its unified content, mathematical modeling, suitability for the integration of technology, active student learning and multiple assessments., which focuses on rates of change. The study investigated an instructional model, using TI-Nspire, specifically designed to promote students' construction of understanding. The document-based feature of TI-Nspire and the display of multiple representations were utilized for teaching and learning at the University Laboratory School during a two-week period from September 10 – September 21, 2007.

The instructional model for using TI-Nspire to introduce a mathematical concept was investigated using Design Research methodology (Cobb, 2003). This is a reiterative process of designing instructional sequences, testing them and making revisions based on use in the classroom and then analyzing student learning so that the cycle of design, revision and implementation can begin again.

The study was designed around the following research question: What effect does the use of TI-Nspire have on scaffolding the development of teachers' questioning skills necessary in an instructional model that builds student understanding? How TI-Nspire becomes an affordance for scaffolding the math understanding and teaching skills of the teacher was an issue addressed in this study. While the skill and knowledge profile of the teacher (and students) determines the degree and type of scaffolding teachers need, this study was designed to provide some insights into teachers' learning curve for using TI-Nspire in inquiry learning and questioning strategies and how this might impact plans for professional development.

This Phase I study is a descriptive study of teaching and learning processes conducted with one teacher and 50 grade twelve students. All 50 students were organized into matched pairs. At ULS students are randomly assigned to one of two groups for mathematics instruction in grade 12. A member from one class was matched to a member in the second class as closely as possible.

Methodology

All students had approximately one-week of experience using TI-Nspire before the start of the research investigation. Thereafter, the two classes participated in the study in a teach-revise-teach cycle as follows: The Treatment Class I of 12^{th} grade students used TI-Nspire for the lessons on developing concepts of rates of change while the Treatment Class II of 12^{th} grade students works on other concepts. After reviewing the assessments and video data from the Treatment Class I, the Project Team revised the lessons for the next teaching iteration, and Treatment Class II were taught the lessons.

Student assessments and videotapes from the Treatment Class II were reviewed to assess whether the revisions were effective in improving learning and teaching.

An instructional design in line with Discovery Learning theory (Bruner, 1967) that is well suited to learning through problem solving and inquiry was developed. This instructional approach allowed us to investigate the role TI-Nspire plays in students' testing of hypotheses and forming generalizations about their findings. Teachers need to provide students with scaffolding so they can develop ways for generating, exploring and organizing claims and evidence to support their mathematical arguments. Since research has found that the best scaffolding is often a mix of social and technological support (Puntambekar & Hübscher, 2005), the project staff provided scaffolding for the 12th grade teacher through pedagogical and content discussions with TI-Nspire. The capabilities of the TI-Nspire learning handhelds provides an opportunity to both scaffold and fade support for learning difficult mathematics concepts and skills. As the project staff engaged the teacher in discussions while providing examples and demonstrations, it was intended that the teacher would engage students in similar scaffolding.

Data was gathered on responses to questions used to assess student understanding. The questions were taken from investigations in Core-Plus Course 4, Unit 1, Introduction to the Concepts of Rates of Change. In addition to student learning assessment measures, videotapes of the classes were used to collect data for qualitative description of the effect of using TI-Nspire in the lessons. The video data informed the revision process and were analyzed for student learning and teacher instructional practice. The video data allowed for examining the critical features of the instructional model such as techniques for presentation, questioning or other activities and feedback.

Procedures

The 12th grade teacher was introduced to TI-Nspire and worked with a member of the project team to learn the features of TI-Nspire and go through some of the activities that were developed to use TI-Npsire during the months of July and August 2007.

July 25, 26, 27, 30, 31, Aug 1, 2	Professional Development for 12th grade teacher to learn Nspire
August 13 - 17	Twelfth grade teacher studied CPM materials to see what to adapt for study
August 19 – September 7	Project team worked with 12th grade teacher to finalize materials for Experimental Lessons – Group 1

TI-Nspire Activities on rates of change were adapted from TI-Npire activities posted on the TI website and used to complement the lessons from the Core-Plus curriculum.

Study schedule:

August 27 – September 7- trained all students in use of TI-Nspire

September 10 – 13 Treatment Class I used TI-Nspire for lessons on rate of change
September 14 - Research Team debriefed and revised instructional model
September 17 – 20 - Treatment Class II used the revised TI-Nspire lessons on rate of change.

Date	Lesson/TI-Nspire Activity
8/21	Algebra Worksheet Introduction to TI-Nspire software
8/22	Algebra Worksheet
8/23	Algebra Worksheet
8/24	Showed video of TI-Nspire on TI website
8/27	Introduction of Nspire Graphing on the TI-Nspire -show different features of Nspire -practice grasp and move feature -notice the change of the functions
8/28	How Fast is the US population growing? -practice data input, statistic, grasp and move feature
8/29	Point, line, slope activity
8/30	Lesson 1 Inv. 1#1 Use TI-Nspire for Inv. 1 #1 -use spreadsheet and graph
8/31	Lesson 1 Inv. 1 #2-4 Take home: Use TI-Nspire to do the "On your own" problem. -review and practice technical kills
9/3	Labor Day
9/4	Lesson 1 Inv. 2 Similarity and difference between average rate of change and instantaneous rate of change Homework: M #1,2; O #1,2,; E #2
9/5	Lesson 1 Inv. 3
9/6	Lesson review Presented Cake problem???
9/7	Quiz Lesson 1
9/10 Treatment – Group A Begins through 9/14	CPM p. 23 Lesson 2 Inv1 #1-3 Tangent Line Problem Adapted from TI website documents CalcAct04_TangentLine_EN and CalcAct04_TangentLine_worksheet_EN Teacher created Student Worksheet ULS_CalcAct04_TangentLine_worksheet_v2_EN

	TI-Nspire document:ULSCalcTangent Line Students worked on 1.1, 1.2 and 1.3 on TI-Npire document Focus teacher conveyed to students: "want to find a formula for rate of change so we can use it for different kinds of function" Teacher summary of lesson: "the concept we wanted to cover today is how the slope gets closer to the curve as q was moved closer to p"
9/11	Teacher's plan for the day as posted on the Elmo: Review Problem # 1 on the activity (10 min) 1.3 on TI-Nspire document Conclude and discuss the Nspire Activity (15 min) Classwork: P 23-25 Lesson 2 Inv. # 1-5 (20 min)
9/12	CMP p.23 Lesson 2 Inv 1 Rate of Change: Linear Functions and Quadratic Functions Went over HW problems # 1-3 together in class TI-Nspire document: Lesson2Inv1
9/13	Review Problems 9 -10 that were assigned for HW Work on Problems 11 – 13 with partners Students shared their work
9/14	Quiz – Bungee Jumper Research Team debrief and revises instructional model
9/17 Treatment – Group B Begins through 9/21	CPM p. 23 Lesson 2 Inv1 #1-3 Tangent Line Problem Adapted from TI website documents CalcAct04_TangentLine_EN and CalcAct04_TangentLine_worksheet_EN Teacher create Student Worksheet ULS_CalcAct04_TangentLine_worksheet_v2_EN TI-Nspire document: ULSCalcTangent Line Students told to finish the first two pages on the worksheet TI-Npire p 1.2 which asked 'what happens to secant as point q is moved toward point p' Focused on zooming in to box a section of the graph Teacher summarized class "we should all have this concept that when we want to find the slope we have to pick two points that are really close"
9/18	Review problem from yesterday Continue to work on p. 3 -4 Homework: Finish Problems # 1- 5 on Investigation 1 on p. 23 Instantaneous Rate of Change
9/19	Went over HW Students shared work in pairs on Elmo
9/20	Rate of change formula for average rate of change and instantaneous rate of change Review problems 9- 10 Work on # 11 – 13

	TI-Nspire document: Lesson2Inv1
9/21	Quiz

Data Analysis

The data analysis was designed to look for patterns and trends from the events of the treatment experiments. Three key criteria related to Design Research theory (Cobb, 2003) were identified for analysis. First, the results from the analysis provided feedback for the instructional design. The two cycles of treatment with a period in between to make revisions served to try out a design, make research-based improvement, and try it out again. The second key feature of the analysis is that it enabled researchers to document the learning of the students as a group. Videotaped classroom data was analyzed to provide a description of a) the teacher's use of questioning to help students develop deeper understanding and b) the level of student knowledge and engagement as evidenced by their mathematical discourse during the lessons. Lastly, the analysis enabled researchers to document individual student reasoning as they participated in the lessons. In addition to the video data, assessment questions from Core-Plus materials were used to determine students' understanding.

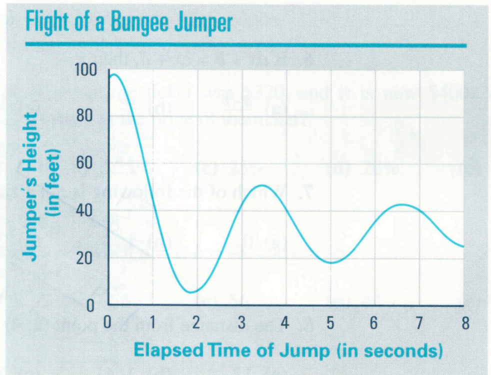
Results

At the end of the period of instruction for both groups, a four-question assessment was given to the students. The assessment was based on a graph of a bungee jumper that was given at the beginning of Lesson 2 in Core Plus along with some Think About It questions. The teacher had not used this in any instruction but in conversations with the Project Team decided that it was a good problem to pose as quiz at the end of the week of instruction. Below is p.22 from CPM that shows how the problem was presented in the textbook materials and following that is the assessment that the teacher and project team developed.

Lesson 2

Rates of Change for Familiar Functions

In the first three courses of the *Contemporary Mathematics in Context* program, you investigated several important families of functions—linear, power, quadratic, exponential, and trigonometric. You found that members of each family had closely-related patterns in tables, graphs, and symbolic rules. In this lesson, you will investigate patterns in the rates of change for these function families. To begin, consider the patterns of change in the height and velocity of a bungee jumper at various points in her flight.



Think About This Situation

Principles of physics can be used to model the flight of a bungee jumper with a rule giving jumper height as a function of elapsed time in flight.

a What type of function model would best fit this pattern of change?

b Suppose you were given the symbolic rule for a function $h(t)$ that predicts a bungee jumper's height at any time t in her jump. How would you use that rule to estimate

- the jumper's velocity at any time?
- the times when the jumper reaches the bottom or top of a bounce?
- the times when the jumper is traveling at maximum speed?

The assessment that was given to students is below:

Rate of change for familiar functions

Name: ______________________________ Date: 9/21/07

The graph in the calculator describes the flight of a Bungee Jumper. The function $f_1(x)$ gives the jumper's height in feet as a function of time in seconds.

1. Calculate the jumper's average velocity ***from bottom of the second fall to top of second bounce up***. Explain your method.

2. Find the average velocity of the jumper in time intervals from x = 1.5 to x = 2. Show your work.

3. Find, as accurately as possible, the jumper's instantaneous velocity at the point (2.0, 5.667). Explain how to obtain your answer.

4. Find what you believe to be the time at which the bungee jumper is falling at the greatest velocity. Give evidence regarding your choice including an explanation why the evidence supports your solution.

For one of the questions, the students' methods of solutions were examined to see if there were differences in the way students approached the solution. The results for these data are discussed below and the actual data are shown in the attached documents.

G1 is used to for the group that had instruction first and G2 is used for the group that had instruction second. The instructor scored each of the four questions on a 'five-point' scale using values, 0, 0.5, 1, 1.5, and 2. A total value for each student was obtained by adding the four individual scores. Q1, Q2, Q3, and Q4 are used to refer to the four questions. The mean valued for the groups per question is given below.

Questions Means	Q1	Q2	Q3	Q4	Total
G1	1.31	1.63	1.54	1.69	6.17
G2	1.39	1.35	1.50	1.46	5.70

The scores ranged from a low of 1.31 to a high of 1.69. G2 had a higher average on Q1 and G1 had higher averages on Q2, Q3, Q4, and the total. For Q2 and Q4 the differences were substantial. Counts of how many of each possible point values were also made. When examining these counts, it is seen that in G1 there was only one student who scored below 1 per question on Q2 and Q4 while in G2 there were 5 students who scored below 1 per question. However, the reverse is true for the Groups for Q1 and Q3. That is, more students in Group 1 than in Group 2 scored below 1 on these questions.

Point Values	**G1 - Number of Students**				
0	1	0	2	0	3
0.5	4	1	2	1	8
1	6	6	2	6	20
1.5	5	3	4	0	12
2	8	14	14	17	53

Point Values	**G2 - Number of Students**				
0	2	3	2	4	11
0.5	2	2	1	1	6
1	5	5	5	3	18
1.5	4	2	2	0	8
2	10	11	13	15	49

On each question the students were asked to show their work or otherwise explain their reasoning. For Q3 there were enough responses from students to be able to attempt to analyze their explanations. Between the groups, 11 different categories of explanations were tallied. The explanations and the total from each group are provided below.

Explanation	**Number in G1**	**Number in G2**
Bracketed equally about x value	11	0
Used x value as one point	5	14
*n*spire only	3	4
*n*spire to verify	1	0
Used trace to find value for y close to given x value but did not use slope	2	0
Confused y value with slope	1	0
Gave velocity for abscissa and ordinate	1	0
treated x coordinate as a and y coordinate as x[or similar]	2	0
Used y value for x in formula	1	0
No Answer/Explanation	0	3
Divided x value by y value	0	2

There are at least two interesting observations from the data in the table above.

1. There is a drastic difference in the first two rows. While either method, bracketing equally about an x-value or using an x-value as one fixed point, no one in G2 used the bracketing procedure. The video is still being analyzed to

see how the difference in instruction between G1 and G2 may have affected students methods.

2. In G1 there six different ineffective strategies used while in G2 there were only 2 [and only 1 if you do not count no strategy as a strategy].

The use of the *N*spire did not differ much between the two, with four in each group indicating use. However we speculate that the use of *N*spire during instruction in G2 led many students to the strategy of “Use x value as one point”.

Appendix

Instructional Materials

How Fast is the U.S. Population Growing?

ID: 8108

Time required
45 minutes

Activity Overview

Students are introduced to modeling exponential data through an investigation of the population of the United States. Students use multiple representations to explore aspects of the growing population. Data was found from the U.S. Census Bureau. The dates run from 1860 to 2006.

Concepts

- *Data representation and interpretation*
- *Visual transformations of exponential functions*
- *Exponential regressions*

Teacher Preparation

This investigation offers opportunities for review and consolidation of key concepts related to exponential, quadratic, cubic, and logarithmic functions. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build a new and deeper understanding.

- *At the Algebra 2/Precalculus level, this activity can serve to consolidate earlier work on exponential functions. It offers a suitable introduction to exploring exponential data, model fitting using exponential functions, and interpretation of graphs.*
- *Begin by reviewing with students the general exponential form of $y = ab^x$.*
- *The screenshots on pages 38–40 demonstrate expected student results. Refer to the screenshots on page 41 for a preview of the student .tns file.*
- ***To download the .tns file and student worksheet, go to http://education.ti.com/exchange and enter "8108" in the search box.***

Classroom Management

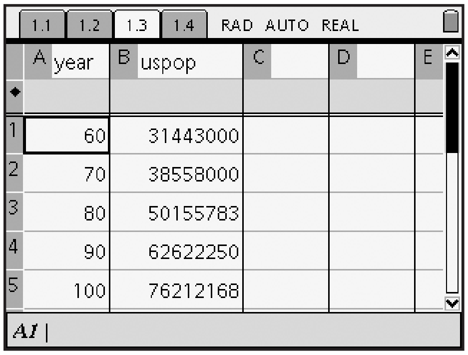
- *This activity is intended to be **teacher-led** with students in **small groups**. You should seat your students in pairs so they can work cooperatively on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *The student worksheet is intended as an investigation through the main ideas of the activity. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *Suggestions for optional extension questions are provided at the end of this activity.*

TI-Nspire™ Applications

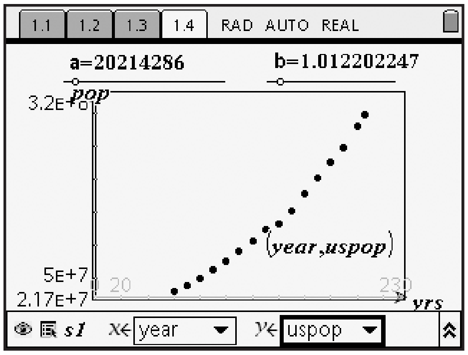
Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes

This investigation explores the growth rate of the United States population from 1860 through 2006. The years have been modified so that “0” represents the year 1800. Throughout the activity, students will develop exponential models to make conjectures based on interpolation and extrapolation of the data.

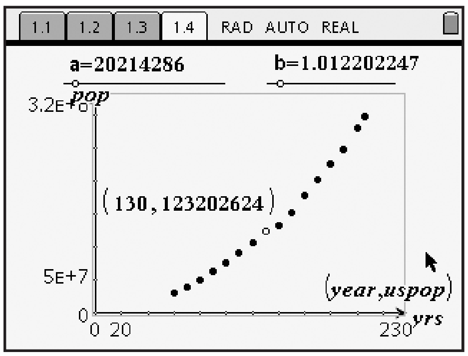
Step 1: Students should first study the data in the spreadsheet on page 1.3 and record any observations on their worksheets. Encourage them to discuss their impressions and observations of the data. You may want to pose questions about what type of function would best model the data.



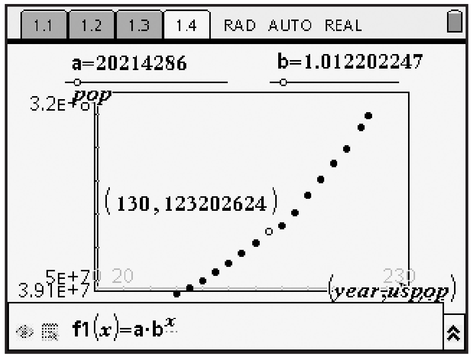
Step 2: On page 1.4, students find a *Graphs & Geometry* application with an appropriate window for this data and pre-constructed sliders at the top of the page. Instruct students to create a scatter plot of the data by selecting **MENU > Graph Type > Scatter Plot**. The function entry line appears and students should press enter to open the drop-down menu for the *x*-variable—select **year**. Press tab to move to the drop-down menu for the *y*-variable—select **uspop**.



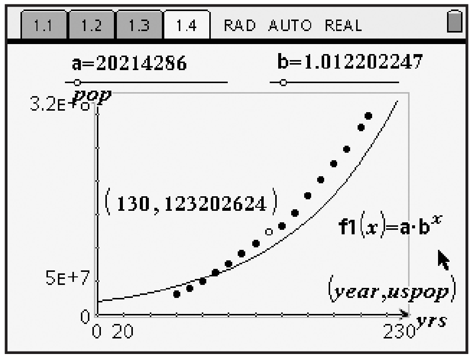
Step 3: Tell students to select the **Point On** tool ( ) from the Points & Lines menu and click on a data point—this will allow them to trace the data. Select **MENU > Tools > Attributes** to change the appearance of that point to be an open circle (to increase its visibility). If the *y*-coordinate appears in scientific notation, instruct the students to hover the cursor over it and press the key to display the number in standard notation.



Step 4: Direct students to press ctrl + G to unhide the function entry line. They should then select **MENU > Graph Type > Function** to change graphing modes, key in "*a* · *b*^{x}" for **f1**, and press enter. (Note: It is important that students enter the multiplication symbol between the pre-defined variables *a* and *b*. Otherwise, the device searches for the variable named "*ab*.")



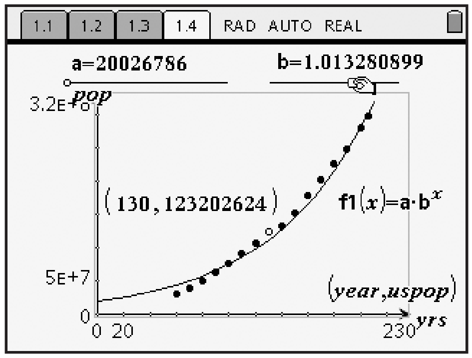
Step 5: The graph of the function and the function rule now appear on the screen. Suggest that students press ctrl + G once again to *hide* the function entry line. They may also grab and drag text boxes and rearrange the screen so as to provide more organization and better visibility.



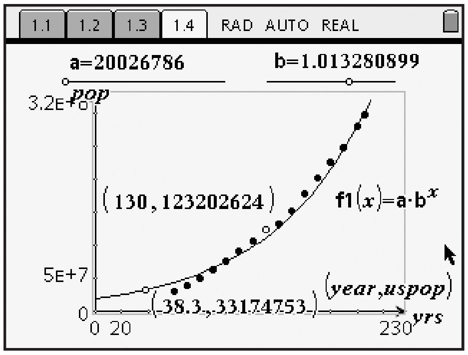
Step 6: Instruct students to grab and drag the points on the **a** and **b** sliders until they have a visually "good fit" of the data. One sample equation is given by:

$$\mathbf{f1}(x) = 20{,}026{,}786 \cdot 1.013280899^x$$

This would be a good opportunity to discuss why they choose the values they did to get the fit they have.

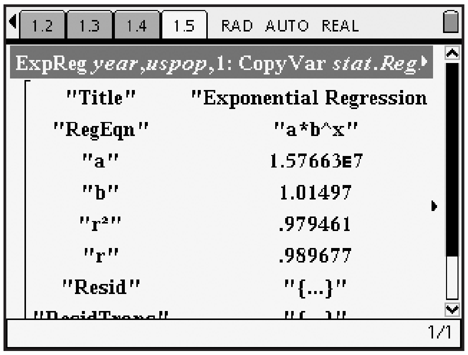


Step 7: Tell students to once more use the **Point On** tool—this time placing a point on the function. Again, students may wish to use the **Attributes** tool to change the point's appearance and press the key while hovering over the point's coordinates to adjust the display digits.

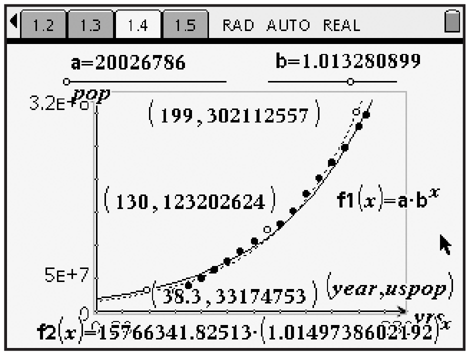


Step 8: Next, have students insert a new page by pressing ctrl + I and selecting **Add Calculator**. Students should be directed to perform an exponential regression by selecting **MENU > Statistics > Stat Calculations > Exponential Regression**.

To set up the regression, tell them to select **year** as the X List and **uspop** as the Y List. (Press tab to move between the drop-down menus.) Make sure the regression equation is set to be saved as **f2** and press enter to compute the regression.



Step 9: Now students should return to page 1.4, unhide the function entry line (ctrl + G) and press ▲ on the NavPad to access **f2** if needed. Tell them to press enter to display its graph, and change its **Attributes** so it appears as a dashed line. Now direct them to repeat Step 7 above: placing a point on this function, thus enabling it to be traced, altering is attributes and coordinates as desired.



Students now have the information they need to explore the U.S. population! Have them complete the exercises on the student worksheet, or use those questions to foster a whole-class discussion.

Solutions – Student Worksheet Exercises

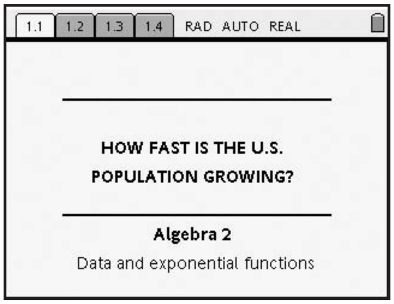
1. Answers will vary. Accept reasonable explanations.
2. ≈1.5%; Possible answer: In exponential equations of the form $y = a(1 + b)^x$, b represents the percent of increase. In the regression equation, $1 + b \approx 1.015$, so $b \approx 0.015$, or 1.5%.
3. a = population of the U.S. in 1800; Answers will vary. Accept reasonable explanations.
4. population in 2020 → $x = 220$; **f2**(220) ≈ 414,762,602; Accept reasonable predictions for **f1**.
5. population in 1776 → $x = -24$; **f2**(−24) ≈ 11,036,068; Accept reasonable estimates for **f1**.

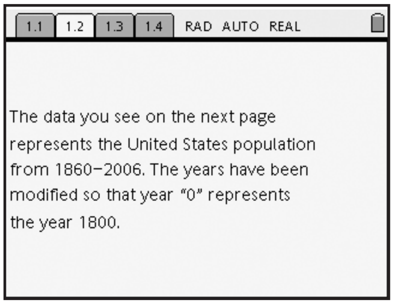
Optional Extension Questions

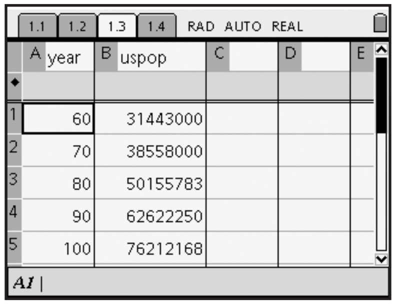
1. Use residuals and Σ^2 to compare the two models.
2. Use other types of functions to model the data.
3. Gather data for the populations of India or China and compare initial values and growth rates with that of the U.S.
4. Compare population versus land mass.

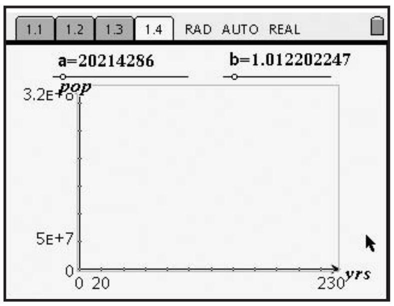
How Fast is the U.S. Population Growing? – ID: 8108

(Student)TI-Nspire File: *Alg2Act1_USPopulation_EN.tns*









How Fast is the U.S. Population Growing?

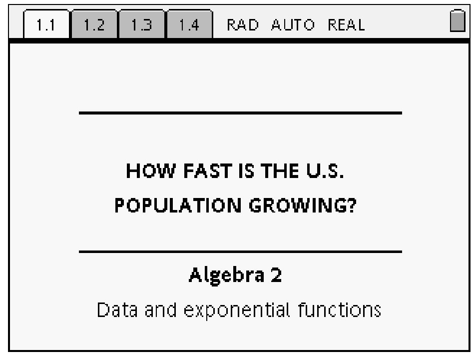
ID: 8108

Name ______________________

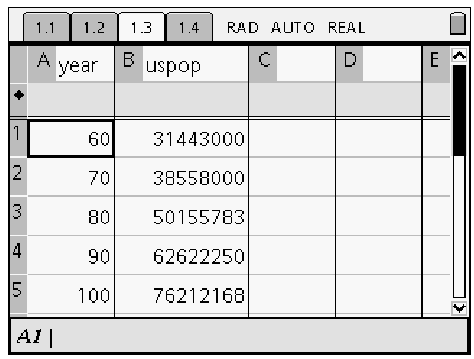
Class ______________________

In this activity, you will explore:

- *trends in population data*
- *modeling data with exponential functions*
- *make conjectures from data*

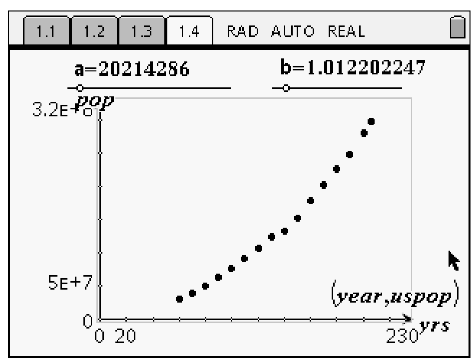


Open the file *Alg2Act1_USPopulation_EN.tns* on your handheld and follow along with your teacher to work through the activity. Use this document as a reference and to record your answers.



Read page 1.2 to familiarize yourself with the activity. Then look at the data in the spreadsheet on page 1.3.

- Describe any trends you see in the data.



On page 1.4, follow your teacher's directions to make a scatter plot of this data. The year (after 1800) will be plotted on the horizontal axis and the population on the vertical axis. When you have finished, it should resemble the one shown here.

Listen as your teacher tells you how to perform the rest of the task of this activity. You will be modeling the data with exponential functions of the form $y = a \cdot b^x$, first by dragging the sliders located at the top of the screen (this will be **f1**), and then by performing an exponential regression of the data (this will be **f2**). You will also place points on the graphs of these two functions, which you can drag along the function, viewing its coordinates.

Before moving to the exercises, first sketch a curve of best fit through the points on the scatter plot shown above, and record your functions **f1** and **f2** in the appropriate space below.

f1(x) = ______________ **f2**(x) = ______________

Exercises

1. Compare your equations for **f1** and **f2**.
Describe any similarities and/or differences you see.

2. Consider the function **f2**. By what percent does the population of the United States increase each year according to this exponential model? Explain.

3. Consider the function **f1**. What does the variable *a* represent in this real-world context? What is the value of *a* according to this model? Does this seem reasonable? Explain.

4. Predict the U.S. population in the year 2020 algebraically using both **f1** and **f2**. Show your work. Grab and drag the points on the two different equations to confirm your results.

5. Estimate the United States population in the year the *Declaration of Independence* was signed (1776), again using both **f1** and **f2**. Show your work. Grab and drag the points to confirm your results. You may have to adjust your window settings.

The Tangent Line Problem

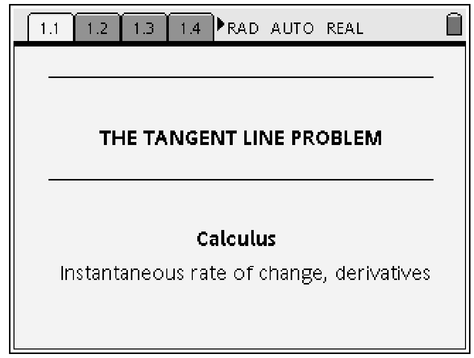
ID: 8315

Name ______________________

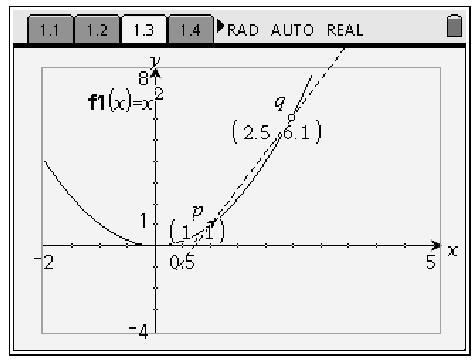
Class ______________________

In this activity, you will explore:

- *the slope of a tangent line*



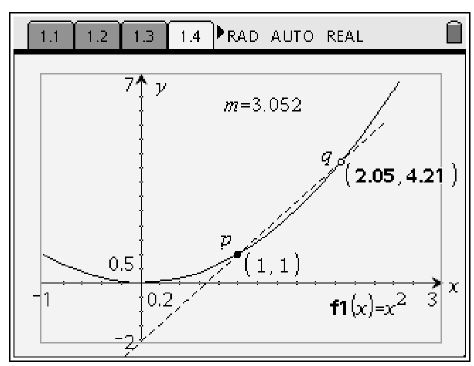
Open the file ULS_*CalcAct04_TangentLine_EN.tns* on your handheld and follow along with your teacher for the first two screens. Use this document as a guide to the activity and a place to record your answers. Move to page 1.2 and wait for further instructions from your teacher.



Consider the diagram shown on your calculator screen. Use the coordinates of points *p* and *q* to determine the slope of $\overleftrightarrow{pq}$. Now imagine that you start moving point *q* towards point *p.* What do you think will happen to the slope of $\overleftrightarrow{pq}$?

What will happen to the slope of $\overleftrightarrow{pq}$ when *p* and *q* become the same point?

Problem 1 – Investigating the slope of a tangent line graphically



Advance to page 1.4. You will see the screen at right showing the graph of the function $f(x) = x^2$ along with secant $\overleftrightarrow{pq}$. The slope, *m*, of $\overleftrightarrow{pq}$ is also displayed on the screen.

Step 1: Drag point *q* slowly towards point *p* and observe the effect on the slope.

1. As you moved point *q* towards point *p*, what value did the slope, *m*, approach?

Continue to answer the questions on following page.

2. In the introduction to this activity, use the formula for slope, $m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$, to find the slope of the secant for *p*(1, 1) and *q*(2.5, 6.1).

3. Why is it <u>not</u> possible to use this formula when point *p* and *q* coincide?

4. How can you estimate the value of the slope of a tangent line at a specific point (example: at x = 1) using the slope formula?

- **STEP 2**: Test your conjecture by advancing to page 1.5 and setting up an expression that will yield the approximate value of the slope of the tangent line to $\mathbf{f_1}(x) = x^2$ at x =1. (Do the calculation on the bottom part of the screen)

Slope of the tangent line between x = ______ and x = _________.

$$\frac{f_1(\quad) - f_1(\quad)}{(\quad) - (\quad)} =$$

Slope of the tangent line between x = ______ and x = ________. (use a closer point than above)

$$\frac{f_1(\quad) - f_1(\quad)}{(\quad) - (\quad)} =$$

Now, using the same expression to find the slope of the tangent line at x = 3.

$$\frac{f_1(\quad) - f_1(\quad)}{(\quad) - (\quad)} =$$

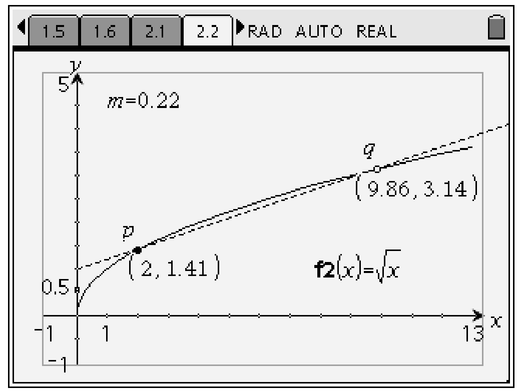
5. The slope of the tangent line is often referred to as the *instantaneous rate of change*. Explain what is meant by this.

Problem 2 – A look at another tangent problem

STEP 3: Now let's try to find the slope of a tangent to $\mathbf{f_2}(x) = \sqrt{x}$ at $x = 2$. Advance to page 2.1 and set up an expression that will yield the approximate value of the slope of this tangent line and record your answer here.

$$\frac{f_2(\quad) - f_2(\quad)}{(\quad) - (\quad)} =$$

STEP 4: Confirm your answer by advancing to page 2.2 and dragging point *q* towards point *p* until the two points coincide. Is the value of the slope on this screen the same as the value you calculated?

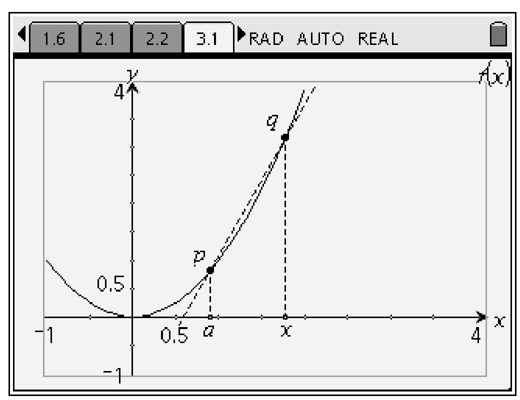


STEP 5: To zoom-in between two points: Press **MENU>WINDOW>ZOOM-BOX.** Press **Click** to select the left corner of the box, then drag to the right corner and press **Click>ESC** to select a region. Continue dragging point *q* towards point *p* and repeat zoom-in process until the two points almost coincide. ***What is the value of m?***

It is the value of the rate of change of to $\mathbf{f_2}(x) = \sqrt{x}$ at $x = 2$.

Problem 3 – Looking at slopes of secants and tangents symbolically

STEP 6: Advance to page 3.1. Write a general expression that will give the slope of a secant line through points *p* and *q*. Use *a*, *x*, and function notation to write this expression.

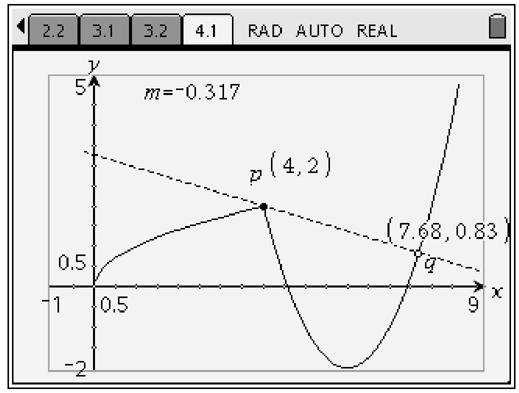


For $y = f(x)$, the rate of change at point *a* is equal to

$$\frac{f(\quad) - f(\quad)}{(\quad) - (\quad)}$$, where *x* is very close to *a*.

Problem 4 – Is there always a tangent line?

STEP 7: Advance to page 4.1. You will see a continuous, piecewise defined function *f*(*x*), with a secant drawn through points *p* and *q*. Once again, we are interested in finding the slope, or instantaneous rate of change, at point *p*.



STEP 8:Drag point *q* towards point *p* such that point *q* is moving towards *p* from the right. What value does the slope appear to be approaching?

STEP 9:Now move point *q* to the left of point *p* and start dragging point *q* towards point *p* again, this time from the left. Now what value does the slope appear to approaching?

Based on your observations, what can you say about the instantaneous rate of change at a point where a corner exists?

Think about the type of functions we have learned. Identify a function that also has a corner at a point like the one (on page 4.1) exists above for which the instantaneous rate of change cannot be found?

When the instantaneous rate of change cannot be found at that point, we say the point is ***non-differentiable***.

Rate of change for familiar functions P. 26-27 #11-13
Date: 9/20/07

Name ____________________________

Name ____________________________

Read problem #11 on the book P.26, and work with your partner to answer the following questions.

1. Use the function rule to calculate the water depth when t = 0, 3,9 and 12

Time (hour)	Water Depth (feet)
0	
3	
9	
12	

2. (a) Use the instantaneous rate of change formula to find the rate of change of the water depth in the harbor when t = 10 (show your work and include the correct unit in the answer).

   (b) What is the meaning the answer above?

3. On the graph, drag the point to t = 18, what is the rate of change of the water depth at that point? Interpret what does the negative sign mean in term of the change of the water depth?

4. Study the water depth graph to identify some times when you think the tide will be moving as describe below. Then estimate, as accurately as possible, the rate of change in the water depth at each of those times.

case	**time**	**rate of change**	**The reason for my answer is..**
moving in most rapidly			
moving out most rapidly			
moving in or out most slowly			

Rate of change for familiar functions

Name: ______________________________ Date: 9/21/07

The graph in the calculator describes the flight of a Bungee Jumper. The function $f_1(x)$ gives the jumper's height in feet as a function of time in seconds.

1. Calculate the jumper's average velocity ***from bottom of the second fall to top of second bounce up***. Explain your method.

2. Find the average velocity of the jumper in time intervals from x = 1.5 to x = 2. Show your work.

3. Find, as accurately as possible, the jumper's instantaneous velocity at the point (2.0, 5.667). Explain how to obtain your answer.

4. Find what you believe to be the time at which the bungee jumper is falling at the greatest velocity. Give evidence regarding your choice including an explanation why the evidence supports your solution.

5. In what ways was TI-Nspire useful for understanding the concepts of rate of change?

6. Comment on your experience with using TI-Nspire.

7. What was challenging about using TI-Nspire?

8. In what way did you use TI-Nspire on your own when you took it home?